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Optimal Profit-Sharing Contracts and Investment in an Interest-Free Islamic Economy

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Abstract

In an Islamic financial system, interest is replaced by a profit-sharing system in which risks are shared between lenders and borrowers. Concerns have been expressed that in such a system investment will decline. This paper formulates investment behavior in a profit-sharing system as a principal-agent problem and investigates the relevant issues under conditions of uncertainty and moral hazard. A major conclusion of the paper is that the assertion of investment decline cannot be justified and that, under certain conditions, a profit-sharing system may lead to an increase in investment.

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Summary

In a financial system conforming to the Islamic injunctions against interest rates, business credit is based principally, although not exclusively, on profit-sharing contracts in which the risks are shared between the lenders (savers-investors) and the borrowers (entrepreneurs). Concerns have been expressed that eliminating an ex ante fixed rate of return would lead to a decline in investment. These concerns can be explained theoretically by suggesting that a major reason for the fixed interest rate is that it allows lenders to avoid the costs of monitoring the behavior of the borrowers. Removal of the fixed interest rate from the financial system creates a type of moral hazard problem because monitoring costs may serve as a deterrent to lenders, thus leading to a reduction in investment.

Although much has been written on Islamic banking, no serious attempt has been made to analyze rigorously the implications for economic behavior of adopting Islamic profit-sharing contracts. This paper attempts such an analysis by presenting a model of profit-sharing arrangements between individual investors and firms. Cases of both certainty and uncertainty are considered. An important issue is the observability of profits by the investor when uncertainty is present and there are costs to obtaining information. This paper shows that individual contracts can be designed to take into account the moral hazard problem that arises from asymmetric information on profits. Such contracts may turn out to be optimal. Indeed, in a conventional system, such optimal contracts are being written where some form of moral hazard is present (e.g., wage contracts).

The analysis shows that there is no strong theoretical reason to support the often-made a priori assertion that investment levels would decline if an Islamic profit-sharing system were adopted. To avoid an adverse effect on investment, however, the adoption of an economy-wide profit-sharing system requires the implementation of a legal and institutional framework that facilitates contracting. The Islamic law of contracts provides for such a framework, which has not yet been fully adopted in countries where an Islamic banking system is being established. In the absence of such a framework, monitoring costs could be prohibitive, and investment could consequently be discouraged. On the other hand, the analysis shows that if legal measures are present to safeguard the terms of contracts, investment levels may increase following the adoption of an economy-wide profit-sharing system.

I. Introduction

Despite a growing sentiment in Muslim nations for moving towards a system of economy-wide profit sharing and the fact that two countries have already adopted such a system, no serious attempt has yet been made to analyze in a rigorous manner its implications for economic behavior. 1/ This paper is therefore the first attempt at modeling Islamic profit-sharing financial contracts and analyzing their implications. Models for both the case of certainty and the cases of uncertainty with and without complete information have been presented and analyzed, thus laying the basis for a more informed and rigorous discussion on the subject. It is shown that in a world of perfect certainty and full information, the elimination of interest would have no real consequences. When uncertainty is introduced, however, conditions are derived under which investment would increase or stay the same. Intuitively, the latter result can be explained by noting that when a fixed interest rate is replaced by profit sharing, the constant per unit cost to firms of borrowed capital is removed. Since in the profit-sharing system there are no debtors to the firm, only equity participants, the investors and the entrepreneurs now are all residual income earners. No prior claims therefore need be considered for the calculation of profit. 2/

Since observability has often been regarded as a factor that might preclude the efficient operation of such a system, considerable attention is paid to it here. An investor knows only that he will obtain a certain proportion of profits in return for the investment that he makes. As he may be unable to observe the output or the action of the agent, which influences the level of output, a moral hazard situation arises. Consequently, the investor will seek to contract on the basis of observable factors. For example, if the level of investment is

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1/ Two Muslim countries, the Islamic Republic of Iran and Pakistan, have opted for a comprehensive adoption of an Islamic-based financial system. While Pakistan has chosen a gradual approach towards Islamization, taking eight years to adopt an interest-free banking system in 1985, Iran completely transformed its economy in one move in 1983. Notwithstanding the different approaches, the two countries have faced similar problems. One of these, which is of relevance to this study, is the heavy concentration of bank asset portfolios in short-term trade transactions--probably an outcome of individual risk perceptions. However, this trend, if continued, would threaten longer-term growth prospects.

2/ Since all investors are residual income earners, marginal productivity of capital is not equated to a fixed rate of interest plus one but to one--hence enabling greater profits to be obtained. This can be seen diagrammatically by observing that with a cost of capital, the profit-maximizing level of output is below the point where maximum obtains and the marginal product is one (see Figure 2).

observable and is a good indicator of profitability, the sharing ratio can be made a function of the observed level of investment. Moreover, in the writing of the contract, care would be taken to ensure that within the constraints, both optimal action and optimal amounts of information are elicited from the agent.

The treatment of the moral hazard problem in the manner described above and used in the paper below, is referred to as the principal-agent problem. The agency problem has recently been extensively used for analyzing wage-employment contracting and its implications for the aggregate economy. 1/ To our knowledge this is the first application of the agency theory to the Islamic system of profit sharing. Nevertheless, recent research in economic theory, mainly in the applications of the agency literature, has suggested that the sharing of profit has significant resource allocational and macroeconomic implications. For example, recently Martin Weitzman has suggested that the profit-sharing system is superior to traditional capitalism because a

> profit-sharing system has the potential to automatically counteract contractionary or inflationary shocks--while maintaining the advantages of decentralized decision-making. And these desirable properties are robustly preserved throughout a variety of economic environments. At the very least, widespread profit sharing can be a valuable adjunct to traditional monetary and fiscal policies. 2/

Although Islam manifests a clear preference for an economy-wide application of risk and profit sharing, the attention in Muslim countries has thus far been concentrated in finance and banking applications. There has thus far been little rigorous analysis of the implications of increased uncertainty that would result from elimination of fixed interest and substitution of profit-sharing contracts in its place. 3/

1/ See, for example, Holmstrom (1983), Kihlstrom and Laffont (1983), Grossman and Hart (1983) and Green and Kahn (1983).

2/ Weitzman (1985). Addressing the question of profit sharing as an alternative to the wage system, Weitzman issues a "friendly challenge" for his critics saying "I challenge any one to cook up an empirical real world scenario, with reasonable number of specifications, where a profit-sharing system does not deliver significantly greater social welfare than a wage system."

3/ One such attempt is a doctoral thesis by W.M. Khan (1984) in which he compares debt and equity instruments and, under a given set of assumptions, concludes that where moral hazard is absent, variable rate of return instruments are superior. Arrangements on the basis of fixed rate of return (debt instruments) dominate when moral hazard is present. Khan concludes that one explanation for the dominance of debt instruments in the financial market may be that they minimize the cost of monitoring.

The first section introduces the principal modes of Islamic financing and discusses the status of contracting in Islamic law; it is followed by a section that summarizes the principal-agent and the wage-contracting literature. These are followed by a characterization of the problem under certainty and then under uncertainty with complete information. Finally, the moral hazard problem in Islamic profit-sharing is considered. A summary of the main results of the paper and a discussion of some issues for future research are presented in the concluding section.

II. Financial Contracts and Contracting in Islam

The central feature of an Islamic financial system is the absolute prohibition of interest. Islam, however, encourages trade, which implies the permission to profit thereby. 1/ To facilitate trade transactions Islamic law has developed specific forms of financial arrangements, 2/ the most important of which are called Mudarabah and Musharakah, as principal means of earning profits without resort to charging of interest. In Mudarabah, one party provides the necessary financial capital and the other (the agent-entrepreneur) the human capital needed for performance of the economic activity undertaken. The resulting profit is then shared between the parties in accordance with a sharing rule specified beforehand in the Mudarahah contract. Under such an arrangement, the entrepreneur risks the loss of his time and manpower, but the owner of financial resources bears all the financial losses. Moreover, the entrepreneur is completely free to manage the project undertaken unless otherwise specified in the contract.

Mudarabah traditionally has been applied to commercial activities of short duration. Musharakah, on the other hand, is a form of business arrangement in which a number of partners pool their financial resources to undertake a commercial-industrial enterprise and share in the resulting profit (or losses) corresponding to their share in the financial capital of the enterprise. These profit-sharing arrangements may be applied either to the whole firm or may have project-specific orientation.

Islamic law places a great deal of emphasis on contracts and the necessity for participants to remain faithful to the terms specified in the contract, so much so that faithfulness to the terms of contracts is considered a distinguishing characteristic of a Muslim. The maxim that "Muslims are bound by their stipulations" is recognized by all schools

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1/ Khan (1985).
2/ Khan and Mirakhor (1985).

of Islamic thought. 1/ Throughout the legal and intellectual history of Islam, a body of rules constituting a general theory of contracts--with explicit emphasis on specific contracts such as sales, lease, hire, and partnerships--were formulated based on the primary sources of Islamic Law. 2/ This body of rules established the principle that, in matters of civil and economic dealings, any contract not specifically prohibited by the Law is valid and binding on the parties and must be enforced by the courts, which are to treat the parties to a contract as complete equals. The core notion of contract is understood in Islamic Law as meaning that the rights and duties between the two parties are specifically determined and fixed by their own voluntary and actual agreement. 3/

III. Principal-Agent Theory

An agency relationship arises whenever one individual depends on the actions of another. The individual taking the action is called the agent and the affected party, the principal. In a Mudarahah contract, for example, the owner of the financial capital is the principal and the entrepreneur, the agent. The two enter into a contract to undertake an economic activity. Accordingly, the agent takes an action a whose result is an outcome x, a random variable whose distribution depends on a. The two parties have agreed beforehand that the agent will receive a share, $S(x)$. If we assume x to be the profits resulting from the economic activity, then the principal's share will be $x-S(x)$. The choice of $S(x)$ depends on the attitude of the two parties toward risk and assumptions regarding the extent of the information available to both parties. 4/ The Principal-Agent problem combines the two elements of risk sharing and differential information and has a first-best solution if all information is costlessly shared between the two parties or

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1/ In a very terse, direct and forceful verse, the Quran exhorts "O you who believe, fulfill (your) contracts" and directs Muslims to reduce their contracts to writing and have witnesses to the conclusion of their agreement. The faithfulness to one's contractual obligations is so central to Islamic belief that when the Prophet was asked "who is a believer?" he replied that "a believer is one with whom the people can trust their person and possessions."

2/ By primary sources of Islamic Law we mean the Quran and the actions and sayings of the Prophet which were meant to illustrate, explain, and exemplify the teachings of the Quran.

3/ Consequently, Islamic Law as laid down early in Islamic history defines a contract in a manner quite similar to the modern notion of a contract.

4/ For earlier work on principal-agent models, see Wilson (1969), Ross (1973), and Mirrlees (1976). For a good basic review of the theory see Rees (1985, 1985). For more recent work in this area see Harris and Raviv (1979), Holmstrom (1979), Shavell (1979), Gjesdal (1982), Hughs (1982), Grossman and Hart (1983), and Singh (1985).

if the incentives of principal and agent can costlessly converge. However, such optimal risk sharing is not possible when along with uncertainty the two parties have unequal information, i.e., an information asymmetry exists. Given information assymmetry, a first-best solution is still possible if the principal can monitor the agent's action and obtain information perfectly and costlessly. If not, only second-best solutions are possible. This shortfall is referred to as an agency loss or agency costs.

The Principal-Agent literature has primarily focused on the case in which (1) the agent's action is not directly observable by the principal; and (2) the outcome is affected but not completely determined by the agent's action. If the principal can not observe the action taken by the agent but is able to make some observation, e.g., of the output, the reward or fee schedule can be set in advance stating that the agent's reward will be a function of the observation made by the principal. When the agent makes some observation not shared with the principal (and bases his action on that observation), but the principal cannot determine whether the agent has used his information in the way that best serves the principal's interest, moral hazard arises. 1/ In this case the principal has to determine a contracting rule that will induce the agent to act in an optimal manner. This problem is referred to as incentive compatibility.

The attitudes of the principal and agent toward risk are crucial in the determination of optimal contracts. In general, second-best solutions involving risk sharing would obtain if both parties are risk averse. When the agent is risk-neutral, he bears all the risks. The principal receives a fixed amount while the agent receives the remainder, i.e., $S(x) = x-k$ where k is determined by the participation constraint. 2/ But since all individuals are averse to sufficiently large risks, the simple solution of assigning all the risks to the agent alone fails when risks become large compared with agent's wealth. In the general case of a risk-averse agent, his share will be a function of the outcome in order to provide incentives, and risk will therefore be shared.

The following principles have emerged 3/ from the theoretical literature on the Principal-Agent problem: (1) the agency loss is highest when the incentives of the principal and agent do not converge and when acquiring information is costly; (2) given the costs of

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1/ See Pauly (1968), Zeckhauser (1970), Spence and Zeckhauser (1971). Arrow (1971) observes that in cases where there is possibility of moral hazard there is an advantage in behavior which is not motivated by narrow self-interest, i.e., moral behavior.

2/ A participation constraint is the utility offered to the principal to a contract at least equal to what he could achieve in other activities.

3/ See Pratt and Zeckhauser (1985).

monitoring, an optimal level of monitoring will be maintained by the principal; (3) when monitoring is expensive (or its substitutes are inexpensive), less monitoring (or monitoring of a poorer quality) will take place; (4) in a range of real world situations, effective monitoring of such indicators as output is relatively or fully successful in reducing agency costs even if the agent's information and action cannot be fully monitored; (5) values such as reputation, which could be lost through dysfunctional behavior or threat of legal action, are strong incentives for proper behavior on the part of the participants; (6) when the same action is repeated over time, the effect of uncertainty tends to be reduced, dysfunctional behavior is more accurately revealed, and the problem of moral hazard is alleviated, i.e., long-term relationships develop the stocks of values needed for enforcement and make limited monitoring more effective; and (7) when agency costs are reduced, the benefits are shared by both the agent and the principal, therefore, the principals and agents have a common interest in defining a monitoring and incentive scheme that yields outcomes as close as possible to ones that would be produced if information monitoring were costless. 1/

IV. The Deterministic Case: A Simple General Equilibrium Model

We begin with setting out a simple model of profit-sharing in an environment of perfect certainty. Assuming perfect certainty and competitive conditions, the Islamic system described above requires entrepreneurs (agents) to obtain and use funds from individual savers (principals) in the first period, agreeing to return the funds in the second period for a promised ex ante share in the profits (losses), λ, to be returned along with the borrowed funds in the second period. For simplicity and without loss of generality, we shall assume only one consumer (principal) and one producer (agent). The return in the second period to the investors may therefore be expressed as:

$$(1.1) \qquad r = \frac{\lambda \pi^*}{I^*}$$

where I^* is the optimal level of investment in the enterprise and π^* is the optimal profits obtained at that level of investment. The rate of return per unit of investment is then r. Since in this simple case there is only one consumer (principal) and one entrepreneur (agent), the firm's profit-maximizing problem may be expressed as:

1/ The importance of "moral behavior" in reducing agency costs has been pointed out by Arrow (1971). It must be noted that behavior that would be in compliance with the stipulations of contracts is an ethical-legal requirement underlying the Islamic law of contracts. Adherence to and enforcement of these laws can minimize monitoring costs.

$$(1.2) \quad \pi^* = \max_I f(I) - (1 + r)I = f(I^*) - (1 + r)I^*$$

and I is the total amount of funds borrowed by the entrepreneur (agent). Since the firm operates under competitive conditions, it treats r as the cost of capital. 1/ Maximization of (1.2) yields the usual profit-maximizing condition equating marginal product to the cost of capital, 2/ 3/

$$(1.3) \quad f' = 1 + r.$$

Solving for the input demand function from the profit-maximizing condition yields

$$(1.4) \quad I^{*d} = I(r).$$

From the second order conditions for profit maximization, the investment demand schedule is downward sloping in r. The relevant parameter for our purposes, however, is λ, the profit-sharing ratio. Therefore, substituting (1.1) into (1.4)

$$(1.5) \quad I^{*d} = I\left(\frac{\lambda\pi^*}{I^*}\right) = \ell(\lambda).$$

To determine the effect of an increase in the sharing ratio on investment, differentiate (1.5) to obtain,

$$(1.6) \quad I^{*d} = \frac{dI^{*d}}{d\lambda} = I' \frac{\pi^*}{I^*} \frac{1}{1+\varepsilon}$$

1/ Contrary to the model that follows, this model exhibits naive behavior in that the entrepreneur does not use the knowledge that $r = \frac{\lambda\pi^*}{I^*}$.

2/ As shown later this may not be strictly correct since each firm's cost of capital depends on its own profitability and not on some general index of profitability in the economy. However, in this case of identical firms λ and π will be constant across firms.

3/ Primes indicate derivatives.

where $\varepsilon = \frac{dI}{dr}\frac{r}{I}$ is the elasticity of the investment demand schedule and $\varepsilon < 0$ from the second order condition of (1.3).

From (1.6) it can be observed that:

$$(1.7) \quad I^{*d}_{\lambda} \begin{matrix} < \\ > \end{matrix} 0 \text{ if } |\varepsilon| \begin{matrix} < \\ > \end{matrix} 1 \text{ or } |f''| \begin{matrix} < \\ > \end{matrix} \frac{r}{I}.$$

Hence, investment as a function of the share parameter may be upward or downward sloping depending on the elasticity of investment demand with respect to the cost of capital or the rate of interest.

The consumer's problem is that of determining, out of his first period endowment, a certain amount to be saved in order to be made available for the entrepreneur's investment. In the second period since there is no endowment, the consumer lives off the proceeds from his investment. Given Y, the problem can be stated as follows:

$$(1.8) \quad \max_{C_1, C_2} U(C_1, C_2),$$

subject to the constraints

$$Y - C_1 = I$$

$$(1 + r)I = C_2.$$

The first order condition for this problem is

$$\frac{U_1}{U_2} = 1 + r,$$

hence the supply schedule for investable funds is obtained as

$$(1.9) \quad I^{*s} = g(r) = g\left(\frac{\lambda\pi^*}{I^*}\right) = h(\lambda).$$

Differentiation of (1.9) yields

$$(1.10) \quad \frac{dI^{*s}}{dr} = \frac{(U_{12} - rU_{22})\, I - U_2}{A}$$

where $A = U_{11} - 2rU_{12} + r^2 U_{22} < 0$.

Since investment is foregone consumption in the first period, the first term in (1.10) is an income effect, which is negative, while the second is a substitution effect which is positive. This has been illustrated graphically in Figure 1, where the substitution effect $= AB = -\frac{U_2}{A} > 0$, and the income effect $= BC = \frac{U_{12} - rU_{22}}{A} < 0$. The supply schedule will have a positive slope, i.e., $g'(r) > 0$ and hence $\frac{dI}{d\lambda} > 0$, when the substitution effect dominates the income effect. In general,

$$(1.11) \quad \frac{dI^s}{d\lambda} \gtrless 0 \text{ if } |\varepsilon^s| \gtrless 1; \text{ where } \varepsilon^s = g'(r)\frac{r}{I}$$

Equating I^{*d} and I^{*s} i.e., equations (1.5) and (1.9) yield the equilibrium sharing ratio.

It is not surprising that in a world of perfect information and certainty, it does not matter whether investment transactions take place on the basis of a fixed and predetermined rate on the basis of a profit-sharing arrangement. Firms and agents recognize the availability of a market-determined opportunity cost of capital that is available and known with perfect certainty. Transactions are therefore based on this measure of opportunity cost, which also turns out to be the return to capital. In the case of uncertainty to which we now turn, this would no longer be true, since each firm and investor transacts in an environment where the outcome is not predetermined. The risk that now prevails has to be shared between the lender and the borrower.

V. Introducing Uncertainty: Perfect Information

A convenient way to introduce uncertainty into the problem is to assume that some random factor θ influences the production function. For computational ease in later analysis, it is assumed that this factor is multiplicative. Hence the production function is written as 1/

$$\theta f(I).$$

1/ Θ is a random variable with mean 1 and some standard deviation, say σ. This implies that $E(\theta f(I)) = f(I)$. Thus, for each realization of the state of the world, production is defined by $\theta f(I)$. In this manner, one can refer to Θ as the variable that defines the state of the world for us.

Since the profits of the firm are now simply revenues, i.e., $\theta f(I)$ minus the cost to the firm which is the borrowing of the firm (i.e., I), profits may be represented as $f(I) - I$. These profits are to be shared according to a predetermined sharing ratio λ. Thus, the firm retains $(1-\lambda)(f(I)-I)$, while giving $\lambda(f(I)-I) + I$ to the investor.

Assuming a utility function $V(\ )$ for the entrepreneur (the agent), the combined problem for the economy may now be written as:

$$(2.1) \quad \text{Max } EU\left(Y - I,\ \lambda\theta f(I) + (1-\lambda)I\right)$$

subject to

$$(2.2) \quad EV\left[(1-\lambda)\left\{\theta f(I) - I\right\}\right] \geq \bar{V}$$

The first order conditions, with γ as the Lagrange Multiplier for equation (2.2) are:

$$(2.3) \quad -EU_1 + EU_2\left[\lambda\theta f'(I) + (1-\lambda)\right] + \gamma V'\left[(1-\lambda)\theta f'(I) - 1)\right] = 0$$

$$(2.4) \quad E\left(U_2 - \gamma V'\right)\left(\theta f(I) - I\right) = 0$$

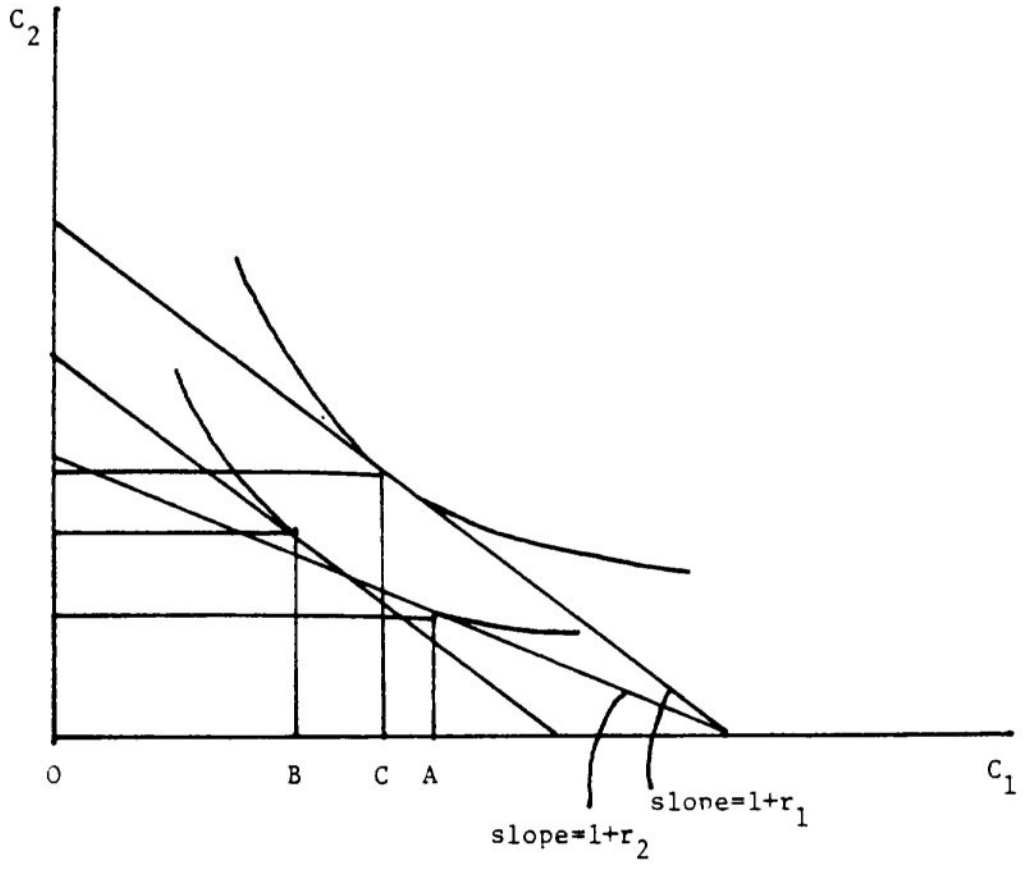
Equation (2.4) implies the Borch condition 1/ of optimal risk sharing, i.e., the ratio of the marginal utilities with respect to λ of the two sharing parties is equal to a constant. This is the usual condition obtained in problems where full information enables optimal risk sharing.

Since in this case the firm borrows up to the point where the marginal productivity of capital is equal to 1 (i.e., $f'(I) = 1$), the third term in equation (2.3) is equal to zero, thus:

$$(2.5) \quad \frac{EU_1}{EU_2} = 1 \text{ for all } \theta.$$

1/ Borch (1962).

Figure 1



$r_1 > r_2$

Consequently, optimal risk sharing enables the consumer to equate expected marginal utilities across time periods. 1/

It is perhaps worth commenting on the first order condition for the firm, i.e., $f'(I) = 1$. This implies that the investment decision of the entrepreneur is independent of the profit-sharing ratio and depends only on the available technology. This result seems plausible since the prevalence of profit sharing leads to a situation where the respective roles of the lender and the entrepreneur are indistinguishable, as both are residual income earners. Unlike the fixed return case, there are no "fixed" costs to borrowing. Nor are there any bond holders whose claims take precedence. It can immediately be seen from the first order condition that investment is larger with profit sharing since in this case, the expected marginal revenue is equated to a constant 1, while in the traditional case, it is equated to $(1+r)$. Diagrammatically, this can be illustrated as in Figure 2. 2/ The ray OC with slope $(1+r)$ represents the cost of capital in the traditional case whereas in the profit-sharing case, OD with slope 1 represents the cost of capital. Consequently, a level of investment equivalent to OA obtains in the traditional case, whereas the profit-sharing case yields a higher level, OB.

VI. Uncertainty and Unobservability

The results obtained thus far have shown that a move from a situation of a known and fixed rate of return to one where only preannounced profit-sharing rules are allowed does not necessarily lead to any observable sub-optimality. Both in the deterministic case and when uncertainty prevails with perfect information, i.e., all participants are able to observe all events as they occur, first-best solutions obtain. Information, however, may be asymmetric because certain events may either not be observable or the cost of observing them may be too high. For example, actions of the agent-entrepreneur that are important to the overall performance of the project undertaken may not be easily observable by the principal, or the production process itself may be subject to some uncertainties and not fully observable by the principal. In these situations there would be a moral hazard problem

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1/ If, as can be expected, a subjective rate of time preference is a part of individual preferences, equating marginal utilities will result in differing levels of consumption in the two time periods. The point can be made clearer by considering the case of separable utility function with an explicit time preference parameter, ρ. Then

$$u'(C_1) = \rho u'(C_2)$$

Thus, with the same utility function for the two time periods differing levels of consumption and hence marginal utilities are obtained because of the time preference parameter.

2/ The illustration assumes certainty for greater clarity.

leading to a second-best solution. In their contracting, however, individual investors would attempt to account for this lack of information. It has been shown in the principal-agent literature 1/ that the principal can use his knowledge of the prevailing uncertainty and the behavioral relationships in the contract in order to obtain optimal behavior from the agent.

A simple way to accommodate asymmetric information is to assume that realization of θ cannot be verified by the principal. In order to overcome the problem of asymmetric information, the individual would now need to write the contract such that all relevant information is used to deduce the state that is realized when, in fact, it is realized. To this end an incentive compatibility constraint, which is essentially like a truth-telling constraint, is utilized. Such a constraint can be written to ensure that in each state the desired level of investment takes place. 2/ The profit sharing then becomes a function of the level of investment which is observable. Using this reasoning the principal's problem can be written as:

$$(3.1) \quad \max_{\lambda(\ )} EU\ (Y - I,\ \lambda\theta f(I) + (1-\lambda)I)$$

subject to

$$(3.2) \quad EV\ \left[(1-\lambda)(\theta f(I) - I)\right] \geq V$$

and $I(\theta)$ is defined by the solution to

$$(3.3) \quad \max_{I(\theta)} EV\ ((1-\lambda)(\theta f(I) - I)),$$

which yields the optimal level of investment for the agent-entrepreneur. The principal, on the other hand, takes this level of investment as given and determines the optimal sharing ratio $\lambda(I)$. The equation (3.3) is the truth-telling constraint which ensures that for a give state, the optimal level of investment is undertaken.

In the standard principal-agent problem where some output which is determined by the agent's action is to be divided, moral hazard is avoided by assuming a risk-neutral agent. 3/ When the agent's effort (which determines the output) is explicitly entered in the principal's

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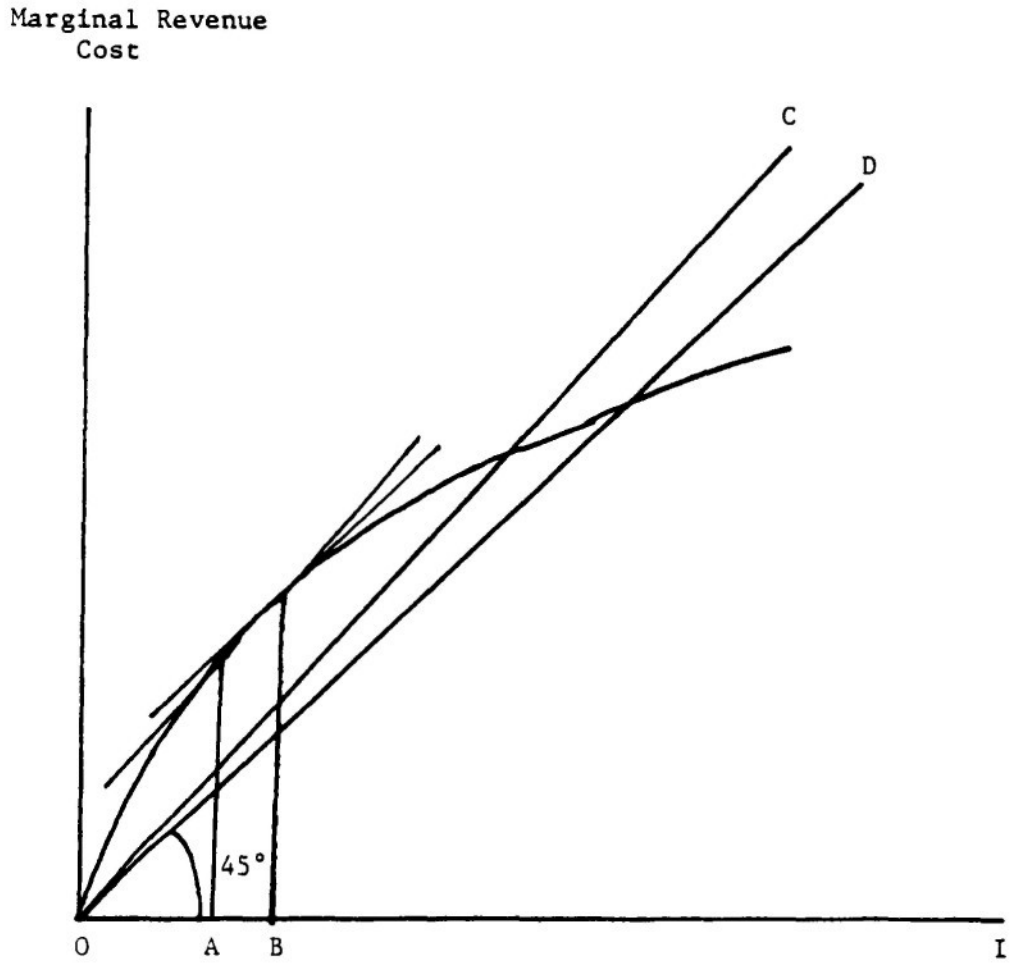
1/ See for example Holmstrom (1972) and Grossman and Hart (1981).
2/ See for example Grossman and Hart (1981) and Green and Kahn (1983).
3/ See for example Harris and Raviv (1976).

Figure 2



utility function, however, moral hazard is not eliminated and the first-best solution does not obtain. 1/ The distinction between standard contracts analyzed in the principal-agent literature where a principal is to share in the output of an agent whose action is unobservable, and those embodying Islamic-type financial transactions where current savings are to be invested at an uncertain rate linked to profitability, were referred to earlier. Additionally, it is more than likely that the providers of investable funds, say investment depositors in an Islamic bank, would be small savers with little opportunity for diversification, whereas the principals are likely to be the banks or entrepreneurs with far greater opportunities for portfolio diversification. For these reasons, in addition to greater tractability, it is assumed here that the agent is risk-neutral.

For the solution of the problem presented in equations (3.1) to (3.3), it is more convenient to work in the state space by making the following transformation: 2/

$$(3.4) \quad k(\theta) = \lambda\theta f(I) + (1-\lambda)I = k(\lambda, I, \theta).$$

It may be noted that $k_I > 0$ and $k_\lambda > 0$ 3/ and hence k can be used, given either λ or I to obtain one or the other as a function of the state variable. Thus, if we can obtain k as a function of the optimal state θ and I as a function of θ we can obtain λ as a function of θ.

Transforming to the state space means that the incentive compatibility constraint should now serve to optimize behavior in each state. Consequently, this constraint in the state space is rewritten such that

$$(3.5) \quad \max_{\tilde{\theta}} \{\theta f(I(\tilde{\theta})) - k(\tilde{\theta})\} \text{ occurs at } \tilde{\theta} = \theta$$

The constraint is introduced in the problem by the statement that the first and second order conditions for that problem hold as identities in θ at $\tilde{\theta} = \theta$. In this manner, optimal behavior by the firm is ensured in all states. The first order conditions which ensure this optimal behavior are

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1/ Green and Kahn (1983).

2/ The formulation of the problem is equations (3.1) to (3.3) is in terms of the sharing ratio λ. The term state space is used to indicate that the transformations allow us to work in terms of each realization of θ, i.e., the state of the world.

3/ Assuming the firm is expected to be profitable.

$$(3.6)\quad \theta f'\left(I(\theta)\right) I'(\theta) - k'(\theta) = 0, \text{ and}$$

$$(3.7)\quad \theta f''\left(I(\theta)\right) I'(\theta)^2 + \theta f'\left(I(\theta)\right) I''(\theta) - k''(\theta) < 0.$$

The problem in equation (3.1) to (3.3) may now be rewritten as

$$(3.8)\quad \begin{cases} \max EU\left(Y - I(\theta), k(\theta)\right) & \text{subject to} \\ E\theta f\left(I(\theta)\right) - k(\theta) \geq \bar{\pi} & \text{and} \\ \theta f\left(I(\theta)\right) I'(\theta) - k'(\theta) = 0 \text{ for all } \theta. \end{cases}$$

A further simplification may be made by letting $f\left(I(\theta)\right) = g(\theta)$ and rewriting the utility function of the principal,

$$(3.9)\quad U\left(Y - f^{-1}\left(g(\theta)\right), k(\theta)\right) = W\left(g(\theta), k(\theta)\right)$$

The derivatives of the two functions are then related as follows

$$U_2 = W_k; \qquad -U_1 = W_g f'.$$

Using g and k as state variables the problem can be written in a considerably simplified form, that is

$$(3.10)\quad \begin{cases} \text{Max } E\, W(g, k) & \text{subject} \\ E(\theta g - k) \geq \bar{\pi}, & \text{and} \qquad \text{and} \\ \theta g' - k' = 0 \text{ for all } \theta. \end{cases}$$

Since the welfare function is concave and the constraints are linear, optimality is guaranteed by the fulfillment of the first order conditions. The Lagrangian expression for the problem may now be written out as 1/

$$(3.11)\quad \int_a^b \left\{W\left(g(\theta), k(\theta)\right) + \gamma\left(\theta\, g(\theta) - k(\theta) - \bar{\pi}\right)\right\} y(\theta) + h(\theta)\left(\theta g'(\theta) - k'(\theta)\right) d\theta$$

1/ In these terms the problem is now similar to the wage contracting problem studied by Green and Kahn (1983). The choice there was between labor and leisure in the same time period whereas in our problem, an intertemporal consumption investment decision is being considered.

where $y(\theta)$ is the density function for the occurrence of the state θ, γ is the multiplier for the profits maximizing constraint and $h(\theta)$ is the functional multiplier for the incentive compatibility constraint. The Euler equations for the problem are:

$$(3.12)\quad y\,(W_g + \gamma\theta) = -\theta h' - h,$$

$$(3.13)\quad y\,(W_k - \gamma) = h',$$

$$(3.14)\quad \theta g' - k' = 0,$$

and the transversality conditions are

$$(3.15)\quad h(a) = h(b) = 0$$

$$(3.16)\quad h(a)a = h(b)b = 0.$$

Under reasonable smoothness and differentiability assumption, unique and continuous functions $h(\theta)$, $g(\theta)$, and $k(\theta)$ will be obtained as solutions. Combining equations (3.12) and (3.13) yields

$$(3.17)\quad W_g + \theta W_k + \frac{h}{y} = 0.$$

Equation (3.17) is the basic result that is derived in most of the principal-agent literature, in that there is a factor that drives a wedge into the first-best solution that would have obtained had there been no problem of unobservability. The factor $\frac{h}{y}$ distorts the first-best sharing rule and determines the direction of the departure from efficiency. In terms of the sharing ratios, the rule can be stated as:

$$(3.18)\quad \frac{-U_1}{f'} + \theta U_2 + \frac{h}{y} = 0.$$

This expression illustrates more clearly the distortionary effects of the moral hazard problem since the first-best solution requires that

$$(3.19)\quad \frac{U_1}{U_2} = \theta f',$$

which is equivalent to

$$(3.20) \quad -\frac{W_g}{W_k} = \theta.$$

Since y is the probability of occurrence of state θ, it is positive. Hence the departure from efficiency is determined by the sign of the multiplier for the incentive compatibility constraint. If $h > 0$, then $W_g + \theta W_k < 0$ implying that

$$(3.21) \quad -\frac{U_1}{U_2} > \theta f'$$

i.e., the marginal rate of time preference that is implicitly derived by the marginal rate of substitution between the two time periods is greater than the marginal product of capital in all states. Consequently, investment levels are greater than they would be in the first-best risk sharing arrangement of the previous section (2.5 in text). Alternatively equation (3.21) shows that the marginal cost of investment as measured by the loss in utility that arises from foregoing consumption in the first period is larger than the marginal return in the second period. Consequently, over-investment (under-investment) will result if $h > 0$ (< 0). Differentiating equation (3.13) with respect to θ and using the incentive compatibility constraint (3.14) yields:

$$(3.22) \quad h'' = (W_{kk}\,\theta + W_{kg})\, g'\, y + y'\left(\frac{h'}{y}\right)$$

and substituting for θ from (3.17):

$$(3.23) \quad h'' = \left[\left(W_{kk}\left(\frac{-W_g}{W_k}\right) + W_{kg}\right) - \frac{W_{kk}}{W_k}\frac{h}{y}\right] g'y + \frac{y'h'}{y}$$

since normality of consumption requires that

$$U_{22}\left(\frac{U_1}{U_2}\right) - U_{21} < 0$$

and since

$U_{22} = W_{kk}$, $U_2 = W_k$; $U_1 = -W_g f'$, $U_{21} = -W_{kg} f'$, and $f'(\theta) = g'(\theta)$, hence

$$(3.24) \quad \left[W_{kk}\left(\frac{-W_g}{W_k}\right) + W_{kg}\right] g' < 0.$$

Thus, the expression in round brackets in (3.23) is negative. Since $g' > 0$, $W_k > 0$, and $W_{kk} < 0$, if $h < 0$, the expression in the square brackets in (3.23) will be negative implying $h'' < \frac{y'h'}{y}$. If $h > 0$ then the relationship $h'' < \frac{y'h'}{y}$ may not hold.

Assuming that $h \leq 0$ at some interior points of [a, b]; since h is continuous, it must attain a minimum in this interval. Call this minimum θ^*. At θ^* then by assumption $h'(\theta^*) = 0$ and $h''(\theta^*) \geq 0$. But this contradicts $h'' < y' \frac{h'}{y}$. Therefore, $h \geq 0$ and in particular $h > 0$ for all θ in the interior of [a, b].

Since h has been proven to be positive, equation (3.17) suggests that $\frac{W_g}{W_k} > \theta$, i.e., the intertemporal marginal rate of substitution is greater than the marginal product of capital. Consequently, investment will be higher than the level that would have prevailed under complete observability when the first-best solution of perfect risk sharing was possible. 1/

In order to see the effect of moral hazard on the profit-sharing ratio, recall that any solution to our original problem defined in (3.1) to (3.3) defines $I(\theta)$ and $\lambda(I(\theta))$ for each θ. These optimal functions for I and λ can be used to define the optimal k and g that will optimize the problem in (3.10). Conversely, now that we have optimal $k(\theta)$ and $g(\theta)$, $I(\theta)$ can be derived by inverting the production function, which is monotonic, to obtain a unique $I(\theta)$ if $I'(\theta)$ is monotonic. Since the incentive compatibility constraint (3.6) holds identically for all θ, differentiating it yields:

$$(3.25)\quad k''(\theta) = f'(I(\theta))\, I'(\theta) + \theta f''(I(\theta))\, (I'(\theta))^2 + \theta f'(I(\theta))\, I''(\theta).$$

1/ Similar conclusion has been obtained in cases where employment contracts based on risk and profit sharing have been analyzed, under specifications similar to those utilized in this paper, using the principal-agent theory, i.e., increases in employment are possible when joint wage-employment contracts are signed (Green and Kahn, 1983). That profit-sharing arrangements have an employment-increasing characteristic has also been emphasized by Weitzman using a different methodology than the standard principal-agent framework (Weitzman, 1983). Weitzman concludes that "there are strong theoretical reasons for believing that were a share system in effect for large firms, the average worker, as well as the economy as a whole, would be better off because of the built-in bias toward eliminating unemployment, expanding production, and lowering prices." The analogy between investable funds as a factor of production as used in this paper and employment needs no elaboration.

Substituting this expression for $k''(\theta)$ in (3.10) the second order condition for the firm's problem becomes

$$(3.26) \quad \theta f'\big(I(\theta)\big)\, I'(\theta) > 0.$$

Since $f'\big(I(\theta)\big) > 0$, $I'(\theta) > 0$. Consequently, for each θ, $I(\theta)$ can be determined uniquely from the optimal $g(\theta)$.

Recall from the definition of $k(\theta)$ in (3.4) that for each k and θ, $\lambda'(I) < 0$. Consequently, since the optimal level of investment in the moral hazard situation is greater than that which would obtain in the first best situation where perfect observability prevails, the return to capital in the moral hazard problem may be less than that in the first-best situation. However, since the level of investment has increased, the total profitability of the enterprise may be increased as observed earlier.

VII. Conclusions

The paper has studied behavior in a system where a profit-sharing rule of borrowing prevails. As discussed, the Islamic system of finance, which is gaining increasing importance in Muslim countries, requires the replacement of the current and traditional system of a guaranteed ex ante fixed rate of return to lenders by a system of profit-sharing between the lender and the borrower. The adoption of this system would therefore result in risk sharing between lender and borrower as well. Consequently, investment behavior and the level of aggregate investment may change as a result of this systemic change. These issues have been examined in the case of certainty and in the case of uncertainty both when information is available to all parties and when there is an observability problem. It has been shown that in the case of perfect certainty and full information, whether investment decisions are based on profit sharing or on a fixed rate of return does not have any real consequences for the economy. When uncertainty prevails, however, the level of investment may actually increase under certain conditions. Intuitively, this latter result seems plausible as the move to a profit-sharing system does away with the distinction between the entrepreneur and the lender. A fixed cost for capital is no longer required to be met as a part of the firm's profit calculations. The marginal product of capital can therefore be taken up to the point where maximium profits are obtained without the constraint of meeting a fixed cost on capital. Both the owners of the firm and the lenders to a firm are now residual income earners.

The paper also serves to illustrate how the view that unobservability may be an externality that precludes the smooth functioning of an Islamic system of profit sharing, may be incorrect. Contracting on the basis of observable indicators that may be correlated with the

unobservable factors could prevent any market failure. Consequently, individual contracting and arbitrage would enable a market-determined sharing ratio for the economy to be determined. Since this form of contracting allows greater utilization of capital, profitability increases. The net return to capital, however, may or may not increase.

The conclusions of this paper are likely to be quite useful in economies that are currently making the transition to a profit-sharing system. First, the results are reassuring in that there is no market failure and individual investors and firms can continue to transact freely and openly in the usual decentralized manner. Second, it illustrates the importance of developing individual firm-specific or project-specific contracts that elicit optimal behavior in the presence of moral hazard. The use of monitoring or sharing rules that are a function of observables, which are in turn correlated with unobservable events or actions, should therefore be encouraged. The efficient working of the system in particular cases would of course depend, to quite an extent, on the ability of the domestic legal system to enforce these contracts in a speedy and judicious manner. 1/

Perhaps the most significant result of the paper is the disproving of the notion that investment levels must decline following the adoption of a profit-sharing system. The paper serves to illustrate that there is no immediately obvious conclusion on the effect of the adoption of profit-sharing system on investment. The models examined here have demonstrated an increase in investment under certain conditions. Before this result can be firmed up for policy purposes, however, further work would be required. First, an examination should be made of the sensitivity of the results that have been derived here to the assumption of risk aversion and to the sources of unobservability. Second, for the effective working of an interest-free Islamic system, efforts will need to be made to limit monitoring, enforcement and contract-design costs. Policy initiative in this direction would require the development of an extensive legal and institutional infrastructure that allows smoother transactions with enhanced monitoring and speedy enforcement. Third, efficiency will further improve if secondary markets develop for the trading of profit-sharing contracts. Although their project- or firm-specific nature would make these contracts imperfect substitutes, such

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1/ An investigation of the process of transformation in the countries adopting an Islamic economic system reveals that bankers ascribe the problem of moral hazard or asymmetric information (real or perceived) to be an important explanation for individual preference for short-term liquidity. This perception of moral hazard is heightened by the unavailability of efficient monitoring systems and the lack of effective legal systems for enforcing contracts. Additionally, but to varying degrees, the lack of a clear definition of private property rights and of the role of private property increases the uncertainty in the environment.

trading would allow a near certain rate of return on capital to be determined. Finally, the issue of adverse selection when profit-sharing contracts are written would need examination.

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