On the Optimality of Sharing Contracts *

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Abstract

A widely held perception among economists is that sharing contracts are inefficient. However, despite their potential inefficiencies and incompleteness, sharing contracts are widely practiced. In this paper, we try to resolve this puzzle. Under state-dependent uncertainty and risk neutrality, we show that sharing can be no less efficient than the first best. By considering bankruptcy cost and stochastic auditing in our model, we show that sharing Pareto-dominates debt, with and without informational asymmetry. Further, by endogenously generating the probabilities in the state-dependent uncertainty, debt ceases to attain efficiency in our model, while sharing preserves its first-best.

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1 Introduction

A widely held perception among economists is that sharing contracts are inefficient. Stiglitz and Weiss (1981), for example, state "in general, revenue sharing arrangements such as equity financing, or sharecropping are inefficient. Under those schemes the managers of a firm or the tenant will equate their marginal disutility of effort with their share of their marginal product rather than with their total marginal product. Therefore, too little effort will be forthcoming from agents". The same problem arises in corporate literature. Harris and Raviv (1991) state "conflicts between shareholders and managers arise because managers hold less than 100% of the residual claim. Consequently, they do not capture the entire gain from their profit enhancement activities, but they do bear the entire cost of these activities".

To elaborate on this argument, suppose the return R on an investment project is an increasing function of the entrepreneur's effort x, such that $R = R(x), R > 0$. Suppose that disutility of effort $C(x)$ is also an increasing function in x, so that $C'(x) > 0$. A rational entrepreneur will choose the optimal level of effort that such that $R'(x) = C'(x)$. Call that effort x^*. Now suppose that the entrepreneur gets only a share α of the return. Her optimal effort now is chosen such that $\alpha R'(x) = C'(x)$. This condition cannot hold at x^*, since $\alpha R'(x^*) < C'(x^*)$. Therefore, effort has to be reduced to $\hat{x}$, such that $\alpha R'(\hat{x}) = C'(\hat{x})$. Since $x^* > \hat{x}$, sharing results in a lower level of effort than first-best solution.

If the project is financed through a fixed-payment contract, like debt, then the entrepreneur has to pay a fixed amount, say r, which does not affect marginal conditions. Thus, a fixed-payment contract is superior in terms of efficiency to a sharing contract.

This argument represents an essential ingredient in models of optimal financial contracts. Another important ingredient is market incompleteness. Moral hazard and adverse selection problems are manifested in the difficulty to observe outcomes and/or efforts, and thus a contract that is contingent on them, such as sharing, is not optimal, while a non-contingent contract such as debt is optimal.

Despite their potential inefficiencies and incompleteness, simple profit sharing incentive contracts are widely practiced. For example, co-ownership (Schmalensee (1989); McAfee and McMillan (1987)), sharecropping (Stiglitz (1974); Eswaran and Kotwal (1985); Ghatak and Pandey (2000)), franchising (Mathewson and Winter (1985); Gallini and Lutz (1992); Bhattacharyya and Lafontaine (1995a)), professional partnerships (Lang and Gordon (1995); Gaynor and Gertler (1995)), and Musharakah and Mudarabah Islamic financing[1] (Ul Haque and Mirakhor (1986);Khan (1989); Presley and Sessions (1994); Ahmed (2002); Ali).

Many studies explain the existence of sharing as tradeoff between its usefulness under certain conditions and its inefficiency. See, for example, models based on risk-sharing properties (Stiglitz (1974), Newbery (1977); Lang and Gordon (1995); see also Rees et al. (1985), and **?**), transaction costs (Murrell (1983); Allen and Lueck (1993)), bargaining powers (Bell and Zusman (1976); Reiersen (2001)), double-sided moral hazard (Reid (1973); Eswaran and Kotwal (1985); Bhattacharyya and Lafontaine (1995b)), moral hazard over choice of project with limited liability (Jensen and Meckling (1976); Basu (1992); see also Sengupta (1997)), joint moral hazard in choice of project and choice of effort (Ghatak and Pandey (2000); De Janvry and Sadoulet (2007)), strategic interaction among principals (Ray (1999)), intertemporal discounting (Roy and

[1] Islamic co-ownership financing instruments are partnership (Musharakah) and trust partnership (Mudarabah). In Musharakah, the partners will contribute capital into the business and share the profit according to an agreed ratio that can differ from the proportion of their initial contribution; however, any loss must be shared according to their initial contribution. Although the ratio of profit sharing should be specified in the partnership agreement, fixing a profit amount is not allowed as it then ceases to be equity financing. Typically, one of the Musharakah partners can be hired as a manager of the co-owned business and may receive a specific fee for her service. The second co-ownership based financing instrument is trust partnership Mudarabah. In Mudarabah, the investor will contribute capital into the partnership and will act as a limited partner while the bank or the investment manager will act as a general partner who contributes work and management. The liability in this type of partnership is limited. The profit will be split according to pre-agreed ratio while losses will be borne by the limited partner. Similar to Musharakah, the profit cannot be a fixed amount. Generally, the general partner is restricted from claiming a salary or a fee for his work but should make his return from profit sharing. Mudarabah usually has an expiration date but can be terminated by either party based on a prior notice. See Usmani (2004) for an introduction of Islamic financial contracts.

Serfes (2001)).

In this study, we take a different rout than in the literature when addressing the optimality of sharing. While many studies start with the presumption that sharing is inefficient relative to the first best and then show how sharing can be optimal in a trade-off model between its inefficiency and benefit (e.g. Stiglitz (1974); Jensen and Meckling (1976); and Grossman and Hart (1982)), we show that under state-dependent uncertainty, sharing can be no less efficient than the first-best solution. By showing this, our approach does not have to depend on any form of a tradeoff to address the optimality of sharing. Additionally, while many studies compare the efficiency of sharing contracts relative to fixed payment contracts, we compare both sharing and fixed payment contracts to self-financing contract, which represents complete market condition.

By taking the more restrictive assumption of risk neutrality, Our results are obtained without having to assume risk aversion as many sharing based models have to usually assume (see Hölmstrom (1979); Holmstrom and Milgrom (1987); Shavell (1979); Sung (1995)).

We also examine our results under market incompleteness. By considering a statedependent profit function with exogenous shocks, bankruptcy cost, and stochastic auditing our model shows that sharing Pareto-dominates debt in terms of expected profits, with and without informational asymmetry. Further, by endogenously generating the probabilities in the state-dependent uncertainty, we show that debt ceases to attain efficient level of effort, while sharing preserves its first-best solution.

The rest of the paper is organized as follows: In Section 2, we review the literature on the optimality of sharing contracts. In Section 3, we present our basic model based on state-dependent uncertainty and complete market. We present our model with symmetric and asymmetric information in section 5 and section 4, respectively. In section 6, we extend our model to assume endogenous probability in the state-dependent uncertainty We conclude in Section 7.

2 Related Literature

The often-labeled Marshallian view[2] argues against profit sharing because an agent exerting effort to generate profit will rationally set her effort in proportion to her profit share[3]. Optimally, profit (and thus effort) will reach maximum when the agent's share reaches one and the principal is compensated with a fixed fee rather than a share of the profit[4]. Taking this view as given , many economists viewed profit sharing as a puzzle and tried to rationalize its existence[5]. For example, in his seminal work, Stiglitz (1974) argues that sharing is practiced by risk-averse agents because of its risk-sharing appeal[6]. Rather than assuming total output risk, agents will share that risk as well as the profit with the principal. Stiglitz (1974) acknowledges that the appealing aspects of risk sharing have to be weighed against the inefficiency of incentives as agents will proportionally reduce their efforts.

Thus, the challenge remains in the literature not in explaining why sharing exists but when it can be optimal. As stated by Allen and Winton (1995): From the viewpoint of risk-sharing, debt seems suboptimal, and costs of financial distress associated with bankruptcy should make debt even less attractive; yet, debt-like contracts are often optimal responses to agency and adverse selection problems. Similarly, although equity has risk-sharing appeal, various institu-

[2]This is named after the analysis in Marshall (1964). The original discussion on the inefficiency of sharing first appeared in Adam smith's (1937) Wealth of Nations in which he condemns sharecropping in agrarian economies and argues for its negative impact on land development.

[3]Although many economists show that sharecropping in agricultural contracts is theoretically inefficient, many empirical studies show similar or even better land productivity and yields levels in sharecropped lands relative to rented lands, see for example, Cheung (1969) in china, Johnson (1950) in the US, Rao (1971) in India.

[4]A notable exception is Cheung (1969) who argues for the optimality of profit sharing as the contract is assumed to stipulate the intensity of using the input as well as the output.

[5]Although most of these studies focus on sharecropping in agrarian economies, their results apply to profit sharing in general. See Singh (1991) for an excellent survey of sharecropping theories.

[6]Newbery (1977) shows that output risk sharing is not sufficient in explaining sharing contracts but the risk of the labour market (factor of production) can lead to optimal sharing contract. Empirical observation also support the argument the risk sharing is not necessary for the emergence of sharing. For example, Allen and Gale (1992) show that likelihood of adopting sharecropping in agricultural contracts in the US is not related to crop variability. Also, Rao (1971) shows evidence that high variance crop in India were rented rather than sharecropped

tional features need to be in place before equity can be used as an effective investment vehicle".

Under standard contracting theory, optimal contracts should be contingent on all relevant information. However, because of incomplete markets, non-contingent contracts such as debt emerges as the optimal in many settings as it ensures that the agent behaves as the principal expects and reports that truthfully (**?**).

Financial contracting literature[7] is rich with different models of optimal incentive-compatible financial contacts. Moral hazard-based models assume that managerial efforts that affect the distribution of outcome are un-observable. In this setting the optimal contracts correspond to debt for investors and equity for managers (Harris and Raviv (1991)). Models based on adverse selection assume un-observability of the borrower type ex ante, rather than the un-observability of ex post efforts or outcomes. The optimal contract under adverse selection is the one that signals the type of the borrower. Allen and Gale (1992) develop a model where distorting earnings is more costly to a good firm than it is to a bad firm, and thus bad firms are more likely to offer securities contingent on earnings. In equilibrium, all firms will offer debt because lenders will deduce that any firm offering a contingent security is a bad firm. Nachman and Noe (1994) develop a model in which the outcome is a function of firm type. In this model, debt is optimal, as good firms cannot signal their quality by increasing payment in bad states and lowering payments in good states.

The above account of the different models used in contacting theory shows to a large extent that debt is optimal under incomplete contacts largely because of moral hazard and adverse selection. Particularly, debt is optimal in financial contracting models because outcomes and/or efforts are not observed and thus cannot be contracted upon. However, it is important to note that the assumptions these various models make in terms of form of incompleteness, risk

[7]For extensive review of financial contacting literature, see Harris and Raviv (1991), Allen and Winton (1995), Hart (2001), and Roberts and Sufi (2009).

attitude, number of contracting periods, and numbers of agents can have an impact on whether debt or sharing is the optimal contract.

Allen (1989) points out to the role of model assumptions in identifying the type of optimal security. In most models one or both agents are risk neutral, and as such risk-sharing is not assumed in the model. However, agents are typically risk averse and efforts and/or outcomes may not be completely unobserved, for example, earnings information. Hölmstrom (1979) suggests that if agents are risk averse and any information on management efforts or outcomes can be observed then the optimal contract should be conditioned on this information. In this case, the debt contract may not be optimal. Holmstrom and Milgrom (1987) and Sung (1995) show that profit-sharing is optimal when an agent with a negative exponential utility function is risk-averse. Shavell (1979) showed that it will not be optimal for the agent (manager) to bear all the risk if she is risk averse, neither would it be optimal for her to bear no risk at all because she will not have enough incentive to exert efforts, and thus risk sharing between the agent and the principal becomes optimal[8]

Early models based on costly state verification find debt to be the optimal contract (Townsend (1979); Gale and Hellwig (1985)) because principles can only verify the actual outcome in different states of the world at a cost. In this setting, the auditing cost makes it suboptimal for investors to verify income every time. They will only do that in bad states of the world when income is below expected levels (in bankruptcy, for example). This type of optimal contract resembles debt as it entails fixed payments in normal states of the world and liquidation under bankruptcy.

However, some model assumptions can drive the optimality of debt. For example, Krasa and Villamil (1994) show that when state verification is random

[8]Utilizing agent risk aversion, Weitzman (1980) went further to calculate an optimal sharing ratio in a contractor-buyer problem that falls somewhere between a cost-plus contract and fixed price contract. Weitzman shows that such optimal sharing ratio depends on uncertainty, risk aversion, and contractor ability to control cost. Berhold (1971) provides a basic structure of profit sharing incentive contract that can be extended to accommodate contracts that are more complex.

the optimal contract entails payments to the lender that increase weakly with returns, and thus does not resemble debt. While the original costly state verification models assume that actual outcomes can be verified ex post at a cost, Lacker and Weinberg (1989) explore the case in which borrowers (or managers) can falsify public records about outcomes at a cost. They show that equity is the optimal contract as it implies no-falsification. Chang (1993) develops a two-period model in which only a manager observes an optimal payout ratio, and diverts the unpaid amount into investments which benefit the manager while giving the investor a below average return in the second period. The optimal contract resembles equity (assuming no costly liquidation) and a combination of equity and debt (assuming costly liquidation).

Reid (1976) builds a model in which the agent and the principal are maximizing their own interest and negotiating the terms of the contracts based on market determined contract parameters. Reid (1976) shows that all contracts (wage, fixed fees, or sharing) can exist and they all are Pareto efficient with different contracts taking place based on the difference in risk premium required by the principal and the agent. The party that requires less risk premium will pear all the risk. Sharing is observed when both parties require equal risk premium. Therefore, sharing under Reid's(1976) model is observed when risk distribution among parties is inconsequential rather than because of risk sharing. Therefore, his argument is valid even under risk neutrality, without the need to assume risk aversion as in Stiglitz (1974).

Unlike principle-agent approach that explains sharing by assuming risk averse agents (Stiglitz (1974)), Some models take a transaction cost approach of Williamson (1979) and Barzel (1997). In these models, risk neutral principal and agent opt for sharing to reduce the monitoring cost over inputs and outputs, see for example Murrell (1983) for multi-period model and Allen and Lueck (1993) for a single period model.

By combining both principal-agent and transaction cost approaches, Agrawal (2002) provides a syntheses of several factors that explain the optimality of sharing mainly risk premia, supervision costs, and shirking. Using a simulation exercise, Agrawal shows that sharing is the optimal contract for a large range of plausible values of supervision costs and risk premia even with persistent shirking.

Other studies explain sharing as natural response to access complementary resources that have no or imperfect markets, such as technical know-how (Reid (1976)), managerial ability (Bell and Zusman (1976)), managerial and supervisory ability (Eswaran and Kotwal (1985)) credit (Jaynes (1984)), labour (Ray (1999)). In these cases, both parties will collaborate with each other to gain access to factors of production owned by the other party. In a similar vein, Reiersen (2001) provide a broad sharing model based on a multistage negotiation. In these cases, the principal will negotiate an incentive contract with the agent, involving him in the production process.

Models based on limited liability also explain the existence of sharing. Basu (1992) and Sengupta (1997) provide models in which the agent, due to limited liability, will not mind taking risk while the principal will be more risk averse in order to reduce the chance of loss. The optimal contract in this case is sharing in order for the principal to gain control over risk. However, these two models did not address the disincentive to exert more effort under sharing contracts. Ghatak and Pandey (2000) provide a model in which two moral hazards are observed, one is in risk taking due to limited liability and the other is in efforts. The model shows that sharing will emerge as the optimal contract under a trade-off between the desire to discourage excessive risk taking by a limited liability agent and the need to incentivize the agent to put more efforts.

Alternative models based on double moral hazard (Bhattacharyya and Lafontaine (1995a); Kim and Wang (1998); Corbett et al. (2005)), or both adverse selection and moral hazard (Guesnerie et al. (1989)) provide the setting for optimality of profit sharing even under risk neutrality. Maskin and Tirole (1999) consider a buyer-seller relationship and show that joint ownership combined with an option for selling the owned share of the asset to the other party can implement first-best incentives. Halonen (2002) considers an infinitely repeated game. In the one-shot game, joint ownership is the worst possible ownership structure, as it minimizes investment incentives but that is not the case under repeated games. Carroll (2015) considers a moral hazard problem in which agent actions are uncertain. In the model, the principal demands robustness and evaluates contracts based on worst-case performance over unknown actions which the agent may potentially take. The optimal contract in such a model involves profit sharing.

Islamic finance literature addresses the design issues of debt versus equity-based contracts. Ul Haque and Mirakhor (1986) utilize a standard principal-agent model in the case of certainty and uncertainty with and without complete information. They show that, similar to debt, profit sharing schemes lead to optimality with full information, under both certainty and uncertainty. Since they were mainly concerned with the aggregate impact of sharing contracts, their results did not provide a complete solution regarding the design of the optimal incentive contract based on profit sharing. Khan (1989) relaxes the risk-neutrality assumption in Ul Haque and Mirakhor (1986) model by assuming a risk-averse capital provider who invests across many managers to spread out the risk. Khan (1989) shows that profit-sharing is Pareto optimal relative to a fixed-return scheme under uncertainty with full information but not under information asymmetry. Assuming risk aversion, Presley and Sessions (1994) develop a model based on Holmstrom and Weiss (1985) and Meyer (1986) in which the outcome of a project undertaken by a single manger is determined by the level of capital investment, the level of managerial efforts and the state of nature. The model shows that with debt, the mangers will be tempted to substitute capital for efforts, which leads to investing less than the first-best level in bad states to maintain incentive compatibility. However, with sharing, the manager, contingent on his contractually specified level of effort, is free to choose the optimal level of investment in each state. In equilibrium, his choice corresponds with the efficient levels, leading to an increase in average investment and a decrease in investment fluctuations. Ahmed (2002) develop an incentive-compatible profit-sharing contract that reduces the moral hazard problem through random audit and a penalty for false reporting. The model also derives the optimal profit-sharing ratio.

Nabi (2015) develops a model to study how an equity based, as opposed to a debt based, financial system would affect capital accumulation and income inequality. in his model, all entrepreneurs share same entrepreneurial skills but they initially belong to one of two groups: the poor and the wealthy. Despite credit market imperfections, Nabi (2015) shows that the poor agents may catch-up with the wealthy agents, causing income inequality to vanish. The rational behind this result is that under equity financial system, the cost of borrowing increases as borrowing from the financial intermediaries exposes wealthy agents to higher sharing ratios. Consequently, the incentive to borrow for expansion purposes gradually vanishes and wealthy agents tend to self finance or become depositors.

The above survey shows that sharing contracts can be optimal under certain conditions and given certain assumptions despite their inefficiency. In this paper, we challenge the inefficiency presumption rather than showing when it can be dominated. Therefore, our paper breaks away from the studies

3 The Model Set Up

We build a simple model to analyze sharing arrangement. We start with an entrepreneur who is able to exert effort x ($x > 0$) to run a certain project for a single period. The project requires an investment of I. If the project is undertaken, it will generate a revenue of R. R depends on two factors: the level off effort exerted and the state of the market. That is, $R \equiv R_s(x)$, where s is the state of the market. There are two states: gain state, denoted as $R_G(x)$, when revenue exceeds investment, i.e. $R_G(x) - I > 0$, and is likely to happen with probability of p and loss states, denoted as $R_L(x)$ when revenue falls short of investment, i.e. $R_L(x) - I < 0$ and is likely to happen with probability of $(1 - p)$.

Under this setting, the uncertainty is modeled as a state-dependent uncertainty rather than a multiplicative (e.g. $R(x) + u$, where u is a random error term) or additive (e.g. $u \cdot R(x)$) uncertainties. In general, additive and/or multiplicative uncertainty can be obtained from state-dependent uncertainty whenever the return function in one state is suitably represented as an affine transformation of the function in the other state. Obviously, this is a restrictive condition, which may or may not hold. Since state-dependent uncertainty is the general case, we expect it to be comparatively more relevant to reality.

All revenue functions are assumed to be concave on effort, i.e. $R_i'(x) < 0$ and $R_i''(x) < 0$ where $i = G$ or L. Moreover, we assume that the marginal gain is greater than marginal loss, i.e. $R_G'(x) > R_L'(x)$.

Given the setting, we define net gain and net loss as

$$G(x) = R_H(x) - I$$

and

$$L(x) = R_L(x) - I$$

respectively. Note that both $G(x)$ and $L(x)$ are also concave on effort and $G'(x) > L'(x)$

Exerting effort create disutility to the entrepreneur, denoted as $C(x)$. $C(x)$ is assumed to be, as in literature, increasing with the amount of effort and convex, that is $C'(x) > 0$ and $C''(x) > 0$. That is, the disutility of extra effort becomes higher if the entrepreneur is already working hard.

3.1 The Case of Self Financing

In this baseline case, we assume that the entrepreneur has sufficient capital to start his business, he chooses effort to maximize his own profit

$$\pi_f(x) = pG(x) - (1-p)L(x) - C(x). \tag{1}$$

If x^* denotes the best level that maximizes 1, then it must solve the first order condition. That is :

$$\lambda(x^*)G'(x^*) = C'(x^*), \tag{2}$$

where

$$\lambda(x) = p\left(1 - \frac{(1-p)\cdot L'(x)}{p \cdot G'(x)}\right) \tag{3}$$

This leads us to the following result,

Result 1. *a self-financed entrepreneur gets a fraction of the marginal return, while he bears the full marginal cost*

Proof. The proof amounts to show that $0 < \lambda(x^*) < 1$. From the first order condition, $pG'(x) - (1-p)L'(x) = C'(x)$. Since $C'(x) > 0$, it follows that $pG'(x) > (1-p)G'(x)$. Since $0 < p < 1$, it follows that $0 < \lambda(x) < 1$ □

This result shows that a self-financed entrepreneur gets a fraction of marginal return, while bears full marginal cost. The optimal effort will always be less than its level under full marginal return. That is, the inefficiency problem exists due to uncertainty even with no sharing of any kind. We argue that sharing arrangements could be designed to achieve the same level of effort of a self-financed entrepreneur, thus losing no efficiency compared to first-best solution.

All we need for this result to be true is that the marginal disutility is positive (more effort is costly), marginal gain is greater than marginal loss and marginal loss to be non-negative (extra effort leads to no or extra loss if market turns out to be bad).

Note that, models assuming either a multiplicative or an additive form of uncertainty would not be able to obtain inefficiency due to uncertainty. Since maximization is performed in terms of expected return, neither form of uncertainty affects marginal conditions, and thus optimal level of input is the same as in case of certainty.

The argument that uncertainty leads to inefficiency is not new to economists. For example, a risk neutral producer would hire less labor under uncertainty than under certainty (McKenna (1986), ch. 4).

4 Financing Under Information Symmetry

Now we study how the entrepreneur would finance the project through external sources under no information asymmetry. External finance could be either in the form of debt or sharing. We start with sharing arrangement then move to debt. We assume that the financier is able to observe the state of the project, i.e., gain or loss, and the level of effort so the entrepreneur cannot misreport revenues.

4.1 Sharing

Here the financier would offer the funding to the entrepreneur in return for a share of the profit in the high state and nothing in the low state. That is, the entrepreneur profit is:

$$\pi_{sh}(x, \alpha) = \alpha \cdot p \cdot G(x) - C(x), \tag{4}$$

and the financier would get

$$v_{sh}(x, \alpha) = (1 - \alpha) \cdot p \cdot G(x) - (1 - p) \cdot L(x) \tag{5}$$

where the subscript sh refer to "sharing". The entrepreneur chooses an effort x in order to maximize his profit subject to the constraint that the financier's profit has to be greater than certain amount that represents her opportunity cost. That is, $v_{sh}(x) \geq \mu$ where μ is the opportunity cost of the fund given to the entrepreneur.

If at optimal effort the constraint is not binding (i.e. $v_{sh}(\hat{x}, \alpha) > \mu$) where $\hat{x}$ is the effort level that maximizes the entrepreneur profit, then the first order condition is

$$\alpha \cdot p \cdot G'(\hat{x}) = C'(\hat{x}). \tag{6}$$

If

$$\alpha = \frac{\lambda(\hat{x})}{p} = 1 - \frac{(1-p) \cdot L'(\hat{x})}{p \cdot G'(\hat{x})},$$

then equation (6) will exaclty be same as the first order condition of a self-financed entrepreneur, i.e. equation (2), and thus the entrepreneur will choose a level of effort same as the one he would choose if she is self-financed. That is, $\hat{x} = x^*$. Note that $\frac{\lambda(\hat{x})}{p}$ is a fraction because as shown in Result 1, $p \cdot G'(\hat{x}) > (1-p) \cdot L'(\hat{x})$.

From now on, we will denote this sharing ratio as $\alpha^* = \frac{\lambda(x^*)}{p}$. We now show that this level of α is the one that is implied by the first order condition of the financier's problem.

Result 2. *If the constraint is not binding, i.e. $v_{sh}(x^*, \alpha^*) > \mu$, then α^* makes x^* to be the optimal level of effort for the financier.*

Proof. The first order condition of the financier problem is $(1-\alpha) \cdot p \cdot G'(x) - (1-p) \cdot L'(x) = 0$. Solving for α, one gets:

$$\alpha = 1 - \frac{(1-p) \cdot L'(x)}{p \cdot G'(x)} = \frac{\lambda(x)}{p}. \tag{7}$$

Thus, if $x = x^*$ then $\alpha = \alpha^*$ and thus the first order condition is met under x^* and α^*. □

That is, under the sharing rule α^*, both profit functions of both, the entrepreneur and the financier, will be maximized at the first best effort. This result points to the symmetry of payoff functions of the entrepreneur and the financier, and that both can be optimized simultaneously. To see this, note that $\pi_f(x) = \pi_{sh}(x) + v_{sh}(x)$. Thus choosing the sharing ratio based on the first-best maximizing conditions leads to maximizing both the financier's and the entrepreneur's profit functions.

If the constraint is binding then, $v_{sh}(x^*;\alpha^*) < \mu$. That is, at the first best level, the financier dose not cover his opportunity cost. However, we can find a sharing rule that covers the financier's opportunity cost and, at the same time, induces the entrepreneur to attain his best level effort. Consider the sharing rule, call it $\hat{\alpha}$, that would make the financier cover his opportunity cost at x^*. That is, $\hat{\alpha}$ that solves $v_{sh}(x^*;\alpha) = \mu$. From equation (5),

$$\hat{\alpha} = \frac{E(x) - \mu}{p \cdot G(x)} \tag{8}$$

where $E(x) = pG(x) - (1-p)L(x)$ and as long as $E(x) > \mu$ then $\hat{\alpha}$ is positive.

If we insert equation 8 into the entrepreneur's profit function in equation (4), we get

$$\pi_{sh}(x) = pG(x) - (1-p)L(x) - \mu - C(x) \tag{9}$$

which has its maximum at x^* (its first order is identical to that of equation (2))

That is, with proper choice of the sharing rule, the first best effort, x^* could always be attained under sharing contract.

The following proposition shows how $\hat{\alpha}$ compares to α^*,

Result 3. $\alpha^* \geq \hat{\alpha}$

Proof. Define $\mu^* \equiv v_{sh}(x^*;\alpha^*)$. When the opportunity cost is just equal to the financier's unconstrained profit function, such that $\mu = \mu^*$, we would have

$\alpha^* = \hat{\alpha}$ or equivalently, $1 - \alpha^* = 1 - \hat{\alpha}$. Using the definitions of α and $\hat{\alpha}$, we get

$$\frac{(1-p)L'(x)}{pG'(x)} = \frac{(1-p)L(x) + \mu^*}{pG(x)}$$

This implies that $L'(x)/G'(x) > L(x)/G(x)$. Now suppose the opportunity cost rises by $h \geq 0$, so that $\mu = \mu^* + h$. In this case the constraint on the financier's profit function becomes binding, and hence:

$$\frac{(1-p)L'(x)}{pG'(x)} \leq \frac{(1-p)L(x) + \mu^*}{pG(x)} + \frac{h}{pG(x)}.$$

Which means that $1 - \alpha^* = 1 - \hat{\alpha} + h/pG(x)$ and hence $\alpha^* \geq \hat{\alpha}$. $\square$

In general, the optimal sharing ratio will be:

$$\min(\alpha^*, \hat{\alpha}). \tag{10}$$

This implies that the financier's profit function will be $v_{sh} = \max(v_{sh}(x^*, \alpha^*), \mu)$.

It is insightful to examine characteristics of α^*. Recall that the entrepreneur's share is

$$\alpha^* = 1 - \frac{(1-p) \cdot L'(x^*)}{p \cdot G'(x^*)}$$

while the financier's share is

$$1 - \alpha^* = \frac{(1-p) \cdot L'(x^*)}{p \cdot G'(x^*)}$$

Note that the financier's share is positively related to expected marginal loss. The greater the risk (or probability) of failure, or the greater the marginal loss, the greater the financier's share. This is intuitive as the financier is the one who bears the losses, so the rise in the financier's share reflects a compensation for the increase in risk he/she bears. Note also that the entrepreneur's share is positively related to his marginal cost of effort. To see this, recall that the first order condition of the entrepreneur is $\alpha^* \cdot p \cdot G'(x) = C'(x)$. Thus, $C'(x)$

equals the nominator of α^*. Hence, the larger the marginal disutility of effort, the larger the entrepreneur's share in gains.

In summary,

- If the financier constraint is not binding, then $\alpha^* = \frac{\lambda(x^*)}{p}$ would make x^* the effort level that maximize both the financier's and the entrepreneur's profit functions.
- If the financier constraint is binding, then $\hat{\alpha} = \frac{E(x^*)-\mu}{p \cdot G(x^*)}$ would cover the opportunity cost of the financier and make x^* the effort that maximizes the entrepreneur's profit.

4.1.1 The Case of Certainty

It is informative to study the implication to the optimal sharing under certainty. Under the optimal sharing ratio, the financier's profit becomes

$$v(x^*;\alpha^*) = (1-p)\frac{L'(x^*)}{G'(x^*)}G(x^*) - (1-p)L(x^*) = (1-p)\left(\frac{L'(x^*)}{G'(x^*)}G(x^*) - L(x^*)\right)$$

Differentiating $v(x^*;\alpha^*)$ with respect to p, we get

$$\frac{\partial v(x^*;\alpha^*)}{\partial p} = -\frac{L'(x^*)}{G'(x^*)}G(x^*) + L(x^*). \tag{11}$$

$\frac{\partial v(x^*;\alpha^*)}{\partial p}$ must be negative as long as $v(x^*,\alpha^*) > 0$, which is the case by assumption.

Thus, a rise in p would reduce the financier's payoff. At some point, the financier' payoff becomes sufficiently small that the constraint becomes binding. From equation (9), the entrepreneur's profit becomes $G(x) - C(x) - \mu$ and the financier's profit is μ. This is a pure debt contract. This confirms the intuition that sharing arrangement is viable only under uncertainty. In case of certainty, sharing is irrelevant, as each party can determine his payoff without any loss of efficiency.

Finally, since $G(x)$ and $L(x)$ are concave, then as probability of success rises, optimal effort x^* also rises, and inefficiency resulting from uncertainty declines. Ultimately, when $p = 1$, optimal effort becomes equal to that in case of certainty.

4.2 Debt

A standard debt contract requires the entrepreneur to borrow capital at the start of the project, then repay the loan together with a fixed interest. The borrower does not provide guarantees for repayment, so it is not a secured debt. In case of success the borrower is expected to get $G(x) - r$[9], where r is the interest rate. In case of failure the borrower is forced to go bankrupt, costing him B. The borrowers expected profit function therefore is

$$\pi_d(x) = p \cdot (G(x) - r) - (1 - p) \cdot B - C(x). \tag{12}$$

The lender gets r in case of success, but loses $L(x)$ in case of failure. His profit is then

$$v_d(x) = p \cdot r - (1 - p) \cdot L(x). \tag{13}$$

The lender demands an amount of interest equals the opportunity cost μ in addition to a risk premium. The financier would chose the interest such that $v_d(x) = \mu$ [10]. the borrower maximizes equation (12) subject to charged interest, i.e. subject to $v_d(x) = \mu$. Solving $v_d(x) = \mu$ for r and substituting into the borrower's profit function, we get:

$$\pi_d(x) = p \cdot G(x) - (1 - p) \cdot (L(x) + B) - C(x) - \mu \tag{14}$$

It is apparent that first order condition for debt financing is identical to that of self financing. Therefore, debt achieves first-best level of efficiency, i.e. at

[9] Note that $G(x)$, the net gain, must be greater that r for the project to be undertaken.

[10] Note that, setting $v_d(x) = \mu$, implies that $r = \frac{\mu}{p} + \frac{(1-p)\cdot L(x)}{p} > \mu$. Thus, r is clearly greater than μ to compensate for the risk of default. Note also that low success probability, p, and high expected loss, $L(x)$, will increase the risk premium, which is very intuitive.

x^*. It is important to note that there is no reason why debt shall be selected in absence of informational asymmetry. As we have seen in section 2, models that obtain optimality of debt do so only in case of asymmetric information. Nonetheless, the analysis in this section will make it much easier to examine the case of asymmetric information later in the paper.

4.3 Comparison

Now we compare sharing and debt in terms of expected profits. We first compare joint profits, then compare profits of each party under the two schemes.

4.3.1 Joint Profit

Here we look at the welfare implication of both schemes.

Result 4. *Expected joint profits under sharing exceed those under debt.*

Proof. Under sharing, joint profit is

$$\Pi_{sh} = \pi_{sh} + v_{sh} = p \cdot G(x) - (1-p) \cdot L(x) - C(x).$$

Under Debt, it is

$$\Pi_d = \pi_d + v_d = p \cdot G(x) - (1-p) \cdot (L(x) - B) - C(x).$$

Since $(1-p) \cdot B > 0$, it follows that $\Pi_{sh} > \Pi_d$. □

The reason joint profits are smaller for debt is positive bankruptcy costs. This is one way the Modigliani and Miller (1958) theorem on equivalence of different forms of finance is violated. Bankruptcy costs are needed to induce the entrepreneur to commit to the project. The presence of such costs therefore helps avoid the problem of lemons, i.e. entrepreneurs who are better off to run away with the financing. While bankruptcy costs have no impact on sharing, they impose dead-weight loss on debt financing.

4.3.2 Pareto Optimality

Let μ^* be the opportunity cost of the financier such that $v_{sh}(x^*, \alpha^*) = \mu^*$, then equation (24) should hold. Under debt financing, the lender's profit is equal to opportunity cost, so we have $v_d = \mu^*$ and equation (23) will hold. Define $E(x) = pG(x) - (1-p)L(x)$ and write

$$\begin{aligned} \pi_{sh} &= E(x^*) - C(x^*) - v_{sh}(x^*, \alpha^*) \\ \pi_d &= E(x^*) - C(x^*) - (1-p)B - v_d(x^*) \\ v_{sh} &= \max(\mu^*, \mu) \\ v_d &= \mu. \end{aligned}$$

Note that

$$\begin{aligned} \pi_{sh} - \pi_d &= (1-p)B - (v_{sh}(x^*, \alpha^*) - v_d(x^*)) \\ v_{sh} - v_d &= \max(\mu^* - \mu, 0). \end{aligned}$$

Now, we are ready for the following result,

Result 5. *Under symmetric information, optimal sharing Pareto-dominates debt as long as* $v_{sh}(x^*, \alpha^*) - \mu \leq (1-p)B$. *Otherwise, neither debt nor sharing Pareto-dominate the other.*

Proof. We know from the optimal sharing problem that $\mu^* \geq \mu$. Consider the following cases

1. If $\mu = \mu^*$, then $v_{sh}(x^*, \alpha^*) - v_d = 0$ and $\pi_{sh}(x^*, \alpha^*) - \pi_d(x^*) = (1-p)B > 0$. Thus, sharing Pareto dominates weakly.

2. If $\mu < \mu^*$ or $\mu = \mu^* - h$, where $h > 0$, then, $v_{sh}(x^*, \alpha^*) - v_d = h$ and $\pi_{sh}(x^*, \alpha^*) - \pi_d(x^*) = (1-p)B - h$. Thus, if $(1-p)B > h$, sharing strongly dominates debt otherwise neither one dominates the other

□

5 Financing Under Asymmetric Information

In this section, we examine the model under asymmetric information. We assume that it is costly to observe effort level as well as the state of the world at the end of the period. We start with sharing, then move to debt.

Before we move to the optimal contracting, in order to allow for moral hazard and the possibility of misreporting, we will assume that within the gain state there are two possible realizations: high gain and low gain, denoted as $R_{HG}(x)$ and $R_{LG}(x)$ respectively. High gain is likely to be seen with probability of q_1 and thus low gain is likely to occur with probability of $(1 - q_1)$. Similarly, within loss states there are also two possible realizations, high loss and low loss, denoted as $R_{HL}(x)$ and $R_{LL}(x)$ respectively. High loss is likely to occur with probability of q_2 and thus low loss is likely to happen with probability of $(1-q_2)$. All revenue functions are assumed to be concave on effort, i.e. $R_i'(x) < 0$ and $R_i''(x) < 0$ where $i = HG, LG, HL$, or LL. Moreover, we assume that marginal gain is greater than marginal loss and that

$$R'_{HG}(x) \geq R'_{LG}(x) > R'_{HL}(x) \geq R'_{LL}(x)$$

Define expected net gain and expected net loss as

$$G(x) = q_1 R_{HG}(x) + (1 - q_1) R_{LG}(x) - I$$

and

$$L(x) = q_2 R_{HL}(x) + (1 - q_2) R_{LL}(x) - I$$

respectively. Note that both $G(x)$ and $L(x)$ are also concave on effort and $G'(x) > L'(x)$.

Given this setting, none of the above results would change. Only the interpretation of $G(x)$ and $L(x)$ would now measure the expected net gain and expected net loss.

5.1 Sharing

Let $\hat{R}$ be the announced net return (net of investment) by the entrepreneur and R_a be the actual return. Since the project might be gaining or losing, $\hat{R}$ could be positive or negative. Untruthful entrepreneur has an incentive to misreport actual return. That is , $R_a \geq \hat{R}$. Given that R_a is not observable to the financier, the entrepreneur has an incentive to misreport for his advantage. In this case, the financier shall communicate a threat of audit to give the entrepreneur an incentive to report truthfully. The financier will audit the reported return (or loss) with a probability of ϕ at a cost of A. If the entrepreneur is audited and found untruthful, he is punished by y.

For any reported return that is less than the $G_H(x^*)$, which will be denoted G_H^* afterward, the financier will not be able to tell whether it is due to misreporting, sub-optimal effort ($x < x^*$) or market conditions, i.e. realizations other than the high gain state (e.g. a loss is reported but G_H or G_L has been realized). All he knows is the gain under the best condition with the optimal effort, i.e G^*. The financier has two situations to consider in order to come up with an optimal threat of audit (optimal ϕ):

1. If $\hat{R} > 0$, R_a will always be a gain and the entrepreneur will be truthful as long as his share of actual return is not less than expected gain from misreporting. That is,

$$\alpha R_a \geq (1-\phi)(R_a - (1-\alpha)\hat{R}) - \phi y \tag{15}$$

2. If $\hat{R} < 0$, R_a might be a gain or a loss and the entrepreneur will be truthful when

$$\max(\alpha R_a, 0) \geq (1-\phi)(R_a - \hat{R}) - \phi y \tag{16}$$

The following result states the optimal audit strategy.

Result 6. *the optimal audit strategy is to set the audit probability ϕ^* such as:*

$$\phi_1 = (1-\alpha)K_1(G_H^*), \quad K_1(G_H^*) = \frac{G_H^* - \hat{R}}{G_H^* - (1-\alpha)\hat{R} + y} \quad if \hat{R} > 0 \tag{17}$$

$$\phi_2 = (1-\alpha)K_2(G_H^*), \quad K_2(G_H^*) = \frac{G_H^* - (1-\alpha)^{-1}\hat{R}}{G_H^* - \hat{R} + y} \quad if \hat{R} < 0 \tag{18}$$

Proof. if $\hat{R} > 0$, then from equation (15), one could easily find that $\phi_1 \geq (1-\alpha)K_1(R_a)$. Since the financier seeks to maximize return, he shall choose the smallest possible value of ϕ to minimize expected costs of auditing. Consequently, ϕ_1 will be set equal to its lower bound, thus $\phi_1 = (1-\alpha)K_1(R_a)$. Both numerator and denominator of K_1 are positive since $R_a > \hat{R}$. Given the fact that $R_a - \hat{R}$ is greater than $R_a - (1-\alpha)\hat{R} + y$, we get $0 \leq K_1 \leq 1$ and in turn $0 \leq \phi_1 \leq 1$. The financier will not be able to tell what is R_a, he just knows $G_H^* \geq R_a$. Noting that $K_1'(R_a) > 0$, then setting $R_a = G_H^*$ in $K_1(R_a)$ ensures that ϕ_1 will always kill the incentive to cheat for all possible values of R_a.

If $\hat{R}$ is loss, then from equation (16), it is clear that $\phi_2 \geq (1-\alpha)K_2(R_a)$. To minimize the cost of auditing $\phi_2 = (1-\alpha)K_2(R_a)$. Since $\hat{R} < 0$, both numerator and denominator of $K_2(R_a)$ are positive and given that $(1-\alpha)R_a - \hat{R} \leq R_a - \hat{R} \leq R_a - \hat{R} + y$, it turns out that $0 \leq \phi_1 \leq 1$. Noting that $K_2'(R_a) > 0$, then setting $R_a = G_H^*$ in $K_2(R_a)$ ensures that ϕ_2 will always kill the incentive to cheat for all possible values of R_a. □

Assuming the formula of optimal auditing is known to both parties, the entrepreneur can predict a priori what would be the probability of audit if he decides to misreport returns. After announcement, the financier computes optimal auditing probability, and implements accordingly. Given such optimal threat of audit, the financier's profit function becomes

$$v_{sh} = (1-\alpha)pG(x) - (1-p)L(x) - \phi^* A \tag{19}$$

and as before, this has to be grater than the opportunity cost μ. On the other hand, the entrepreneur's profit function will be

$$\pi_{sh} = \alpha pG(x) - C(x) \tag{20}$$

Note that ϕ^* is evaluated at x^*, so it is treated as a constant rather than as a function of x. Therefore, as it was shown in section 4.1, α^* and x^* will maximize the profit of both. Thus, under the "optimal threat of auditing", efficient effort (e.g.x^* same as the effort a self-financed would exert) can be achieved, and thus first-best solution is attained even under asymmetric information [11].

5.2 Debt

Under debt financing, the lender needs to audit only when the borrower announces that he is unable to repay the loan plus interest, in which case the borrower is forced to default. As noted earlier, debt becomes optimal under asymmetric information and deterministic auditing. The intuition is simple: since auditing is costly, the lender prefers to minimize auditing. The lender therefore asks for a fixed repayment, thus no auditing is needed. To deter the borrower from falsely announcing inability to repay, non-repayment forces the borrower into bankruptcy. Since bankruptcy costs are sufficiently high, this assures that it is in his interest to repay.

[11] Such a random audit strategy, as this has been shown to be superior to deterministic audit. Intuitively, randomness creates better incentives for the entrepreneur to perform optimal effort. Deterministic auditing, as mentioned in the introduction, is an essential ingredient for the result of optimality of debt. Williamson (1986) admits that he was not able to prove that debt is still optimal if stochastic monitoring is allowed. According to Freixas and Rochet (2008), "standard debt contracts can be dominated if the situation allows for stochastic auditing procedures". This has also been shown by Krasa and Villamil (1994) among others.

The financier's payoff under loan contract will be[12]

$$v_d(x) = pr - (1-p)(L(x) + A) \tag{21}$$

As before, the lender demands an amount of interest equals to the opportunity cost in addition to a risk premium. Charged interest will be such that $v_d = \mu$, where μ represents, as before, the opportunity cost for the financier. The borrower maximizes

$$\pi_d(x) = p(G(x) - r) - (1-p)B - C(x) \tag{22}$$

subject to the charged interest, i.e. $v_d(x) = \mu$.

Solving $v_d = \mu$ for r and substituting into the borrower's profit function, we get

$$\pi_d = pG(x) - (1-p)(L(x) + A + B) - \mu - C(x) \tag{23}$$

First order condition is identical to that in case of symmetric information, so debt also achieves first-best effort despite informational asymmetry. The additional cost of monitoring affects total profit but not marginal conditions.

5.3 Comparison

We now compare expected profits under the two schemes. First we start with joint profits, then examine one-to-one payoffs.

5.3.1 Joint Profit

To make the comparison, we set the financier's profit function so that $v_{sh} = \mu$. Thus, equation (20) becomes:

$$\pi_{sh} = pG(x) - (1-p)L(x) - \mu - \phi^* A - C(x) \tag{24}$$

[12] Again, we assume all gain states to result in gain higher than the interest.

The joint profit in case of sharing is $\pi_{sh} + v_{sh}$.

Using equations (19) and (20) one gets

$$\Pi_{sh}(x^*, \alpha^*) = \pi_{self}(x^*) - \phi^* A \quad (25)$$

in case of debt, the joint profit is the sum of equations 21 and 22

$$\Pi_d(x^*) = \pi_{self}(x^*) - (1-p)(A+B) \quad (26)$$

With this, the following result is obtained

Result 7. *At the optimal sharing ration α^*, the joint profit under sharing is greater than that under debt financing.*

Proof. Note that $\Pi_{sh} - \Pi_d = (1-p)B + (1-p-\phi^*)A$. For this to be positive, $(1-p-\phi^*) \geq 0$. Recall that $1-\alpha^* = (1-p)L'(x^*)/pG'(x^*)$. Since expected marginal gain $G'(\cdot)$ is greater than marginal loss $L'(\cdot)$ by assumption, then $1-\alpha^* < 1-p$, from which it follows that $\alpha^* > p$. From equations (17), we know that $\phi^* = (1-\alpha^*)K_i$, where $i = 1$ or 2. Since, K_i are less than one, it follows that $\phi^* \leq (1-\alpha^*) < 1-p$. Thus, $1-p-\phi^* \geq 0$ □

This result shows that expected joint profits under sharing exceed that under debt, even after costs of monitoring are included. This is attributed to two factors. First, bankruptcy costs affect debt but not sharing. Second, the lender is more likely to audit than the financier, as $(1-p) > \phi*$, so expected costs of auditing are higher under debt. Note that the gap in joint profits between sharing and debt is larger when information is asymmetric than when it is symmetric. The presence of auditing cost represents greater dead weight loss for debt financing but not for sharing. Thus, sharing achieves higher returns under economic imperfections than debt.

We treated bankruptcy costs as equal under the two contracts. However, as argued earlier, minimum bankruptcy costs under debt are higher than those under sharing. So, many projects that could be financed by sharing will be denied capital under debt, a problem known debt overhang (Myers (1977)). Entrepreneurs carrying out these projects are always better off under sharing. The assumption based on which $\phi^* < 1 - p$ is that expected marginal gain exceeds marginal loss, or $pG' > L'$. This implies that probability of success has to be greater than the ratio of marginal loss to marginal gain for sharing to dominate debt. Thus a lower value of p could make debt more profitable. This shows an important difference between the two schemes. Since the financier bears the risk of failure under sharing, he certainly requires the project to be viable, and thus likelihood of success has to meet a minimum level. A lender, on the other hand, does not directly bear such risk, so he is less restrictive in accepting projects with smaller q. In other words, debt financing dominates sharing for less viable projects, but for borrowers with higher bankruptcy costs. This is quite consistent with reality where banks care more for size of the firm than for the profitability of the project financed. Equity providers, like venture capitalists, care more about the viability of the venture than for the size of the firm. Thus the claim that equity tends to support more risky projects is not accurate. In presence of optimal auditing, equity finance can control for risk of fraud. Auditing also has a disciplinary role for the management of the project, which helps the entrepreneur seeks optimal effort needed to maximize returns. Taking these factors into consideration, therefore, projects financed through sharing should be less risky than those financed by debt.

5.3.2 Pareto Optimality

Let μ^* to be the opportunity cost of the financier such that $v_{sh}(x^*, \alpha^*) = \mu^*$ then equation (24) should hold. Under debt financing, the lender's profit is equal to opportunity cost, so we have $v_d = \mu^*$ and equation (23) will hold. Now, the following results shows when sharing is optimal.

Result 8. *Under asymmetric information, optimal sharing Pareto-dominates debt for as long as* $v_{sh}(x^*, \alpha^*) - \mu \leq (1-p)B + (1-p-\phi^*)A$*. Otherwise, neither* *debt nor sharing Pareto-dominate the other.*

Proof. We have:

$$\begin{aligned}
\pi_{sh} &= E(x^*) - C(x) - \phi^* A - v_{sh}(x^*, \alpha^*) \\
\pi_d &= E(x) - C(x) - (1-p)(A+B) - v_d \\
v_{sh} &= \max(\mu^*, \mu) \\
v_d &= \mu
\end{aligned}$$

Note that

$$\begin{aligned}
\pi_{sh} - \pi_d &= (1-p)B + (1-p-\phi^*)A - (v_{sh}(x^*, \alpha^*) - v_d) \\
v_{sh} - v_d &= \max(\mu^* - \mu, 0)
\end{aligned}$$

We know from the optimal sharing problem that $\mu^* \geq \mu$. Consider the following cases

1. If $\mu = \mu^*$, then $v_{sh}(x^*, \alpha^*) - v_d = 0$ and $\pi_{sh}(x^*, \alpha^*) - \pi_d(x^*) = (1-p)B + (1-p-\phi^*)A > 0$. Thus, sharing dominate weakly.

2. If $\mu < \mu^*$ or $\mu = \mu^* - h$, where $h > 0$, then, $v_{sh}(x^*, \alpha^*) - v_d = h$ and $\pi_{sh}(x^*, \alpha^*) - \pi_d(x^*) = (1-p)B + (1-p-\phi^*)A - h$. Thus, sharing strongly dominates debt if $(1-p)B + (1-p-\phi^*)A > h$ but neither one dominate the other

□

Note that the range of strong dominance of sharing over debt has expanded under asymmetric information. Sharing appears more immune to inefficiency and market friction than debt.

6 Endogenous Probability

It is more realistic to view the likelihood of success to be a function of effort. Higher effort can make success more probable, while low effort makes failure more probable. In this case effort has two different effects: one on the probability of success, the other is on the magnitude of gain. Following many models in the literature, we assume probability as a strictly concave, continuous, twice differentiable function of effort:

$$p \equiv p(x), p' > 0, p'' < 0$$

Self-financed entrepreneur therefore chooses effort to maximize

$$\pi_{self}(x) = p(x)G(x) - (1 - p(x))L(x) - C(x). \tag{27}$$

and it can be shown that it achieved its maximum at x_p^* such that

$$\psi(x_p^*)(p'(x_p^*)G(x_p^*) + p(G'(x_p^*)) = C'(x_p^*) \tag{28}$$

where

$$\psi_p(x) = \left(1 - \frac{p'(x)L(x) - (1 - p(x)L'(x)}{p'(x)G(x) + p(x)G'(x)}\right) \tag{29}$$

Note that when $p'(x) = 0$, then $\psi = \lambda/p = \alpha^*$

Result 9. $0 < \psi < 1$

Proof. From (28), $C' > 0$ implies that $\psi > 0$. Thus, $p'G + pG' > (1-p)L' + p'G$. Since all are positive, then $0 < \psi < 1$ and hence $0 < \psi < 1$ □

6.1 Sharing

Assuming the constraint on the financier's profit function is not binding, the entrepreneur's problem is to choose effort to maximize:

$$\pi_{sh}(x, \alpha) = \alpha p(x)G(x) - C(x) \tag{30}$$

Comparing 30 and 28, it is apparent that first-best effort could be achieved if sharing ratio is set to $\alpha = \psi$, so that $x = x_p^*$. Using $\alpha = \psi$, the financier's profit function becomes:

$$v_{sh}(x, \alpha) = (1 - \alpha)p(x)G(x) - (1 - p(x))L(x) \tag{31}$$

It can be easily verified that it achieved its maximum at x_p^* as long as the constraint on the financier's profit function is not binding, i.e $v_{sh}(x_P^*), \psi) < \mu$ choosing $\alpha = \psi$ also maximizes the financier's profits.

6.2 Debt

As before, in debt financing the lender sets his or her return equal to opportunity cost. Thus, $v_d(x) = \mu$ and charged the implied interest. The borrower's objective functions is then

$$\pi_d(x) = p(x)G(x) - (1 - p(x)(L(x + B)) - C(x) - \mu \tag{32}$$

The first order condition becomes

$$p(x_d^*)G(x_d^*) + p'(x_d^*) - (1 - p)L'(x_d^*) + p'(x_d^*)L(x_d^*) + p(x_d^*)B = 0 \tag{33}$$

where is x_d^* is the maximum effort under debt contract. we can state the following property.

Result 10. *With endogenous probability of success, optimal effort under debt* *exceeds that of first-best solution. That is $x_d^* > x_p^*$*

Proof. comparing (33) to (28) we note that

$$\pi'_{self}(x_d^*) + p'(x_d^*)B = 0.$$

which implies that $\pi'_{self}(x_d^*) = -p'(x_d^*)B < 0$ Since the profit function is positively sloped at values less than optimal effort, but negatively sloped afterwards, it follows that $x_d^* < x_p^*$. □

This result is intriguing from risk taking perspective. Additional or excess effort in presence of bankruptcy cost may represent the asset substitution problem raised in Jensen and Meckling (1976). In presence of debt financing, the entrepreneur may be tempted to invest in high risky project which transfers value from the lender to himself. At certain states of nature when the probability of bankruptcy is high, the entrepreneur will exert more effort to increase the chance of success, however those efforts may lead him to overinvest by taking negative NPV projects with a small chance of success but a huge upside potential

6.3 Joint Profit

In case of sharing we have

$$\pi_{sh}(x_p^*, \psi) + v_{sh}(x_p^*, \psi) = \pi_{self}(x_p^*)$$

while in case of debt we have

$$\pi_d(x_d^*) + v_d(x_d^*) = \pi_{self}(x_d^*) - (1 - p(x_d^*))B.$$

Since $\pi_{self}(x_p^*) > \pi_{self}(x_d^*)$, then joint profit in sharing exceeds that in debt by $\pi_{self}(x_p^*) - \pi_{self}(x_d^*) + (1 - p(x_d^*))B > 0$. Thus, There are now two sources of inefficiency in the debt contract: sub-optimal level of effort in addition to bankruptcy costs

6.4 Pareto Optimality

Going through the same analysis as we do in section 4.3.2, sharing Pareto-dominates debt as long as $v_{sh}(x_p^*, \psi) - \mu \leq \pi_{self}(x_p^*) - \pi_{self}(x_d^*) + (1 - p(x_d^*))B$. Otherwise, neither debt nor sharing Pareto-dominate the other.

These results show that with endogenous probability, debt contract is inefficient. Knowing that his effort affects likelihood of success, the borrower becomes under pressure to achieve maximum gain, leading to excess effort. Yet for the relevant range of h, his expected gain is less than that under sharing. Sharing contract therefore economizes on cost of effort meanwhile allows for greater expected gain.

7 Conclusion

In presence of state-dependent uncertainty, marginal conditions deviate from those in case of certainty. Under such form of uncertainty, the classical argument of the inefficiency of sharing arrangements, be it equity financing or crop-sharing, ceases to hold. Sharing achieves first-best efficiency under a variety of conditions under which debt fails to do so. The fact that sharing arrangement is able to achieve first-best efficiency under a variety of conditions point to the flexibility and adaptability of sharing to different environments. Debt, in contrast, suffers from inefficient effort as well as dead-weight loss of bankruptcy costs. Combining endogenous probability and endogenous bankruptcy costs would have conflicting impact on resulting effort. The gap in joint profits between sharing and debt, however, may not necessarily shrink. At best, the term $\pi_{self}(x_p^*) - \pi_{self}(x_d^*)$ might be zero, but the term $(1p(x))B$ would remain.

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