

VOLUME No. 5

INTERPRETING OPTION VOLATILITY

This issue of *Derivations* will unravel some of the complexities of volatility and the role volatility plays in determining the price of an option. We will show you how to use implied volatility to help gauge the market's expectation of future rate variability. Also, we'll look at how a hedge is affected by a change in implied volatility and the influence of a hedge's strike or time to expiration.

THE CHALLENGE

Concern about market volatility is the major force driving all hedging decisions. Volatility is a key determinant of the cost of option-based hedging instruments such as caps, floors, collars and swaptions. In the dealer market volatility is so important to option pricing that most price quotes for option trades are expressed in volatility units rather than dollars.

Volatility is a measure of how much prices or rates have moved or are expected to move. *Historical* volatility represents past price or interest rate movements. *Implied* volatility represents the market's *expectation* of how volatile prices or rates will be in the future. It is called implied because it cannot be directly observed, only inferred from the price of the option when all other variables affecting the price are specified.

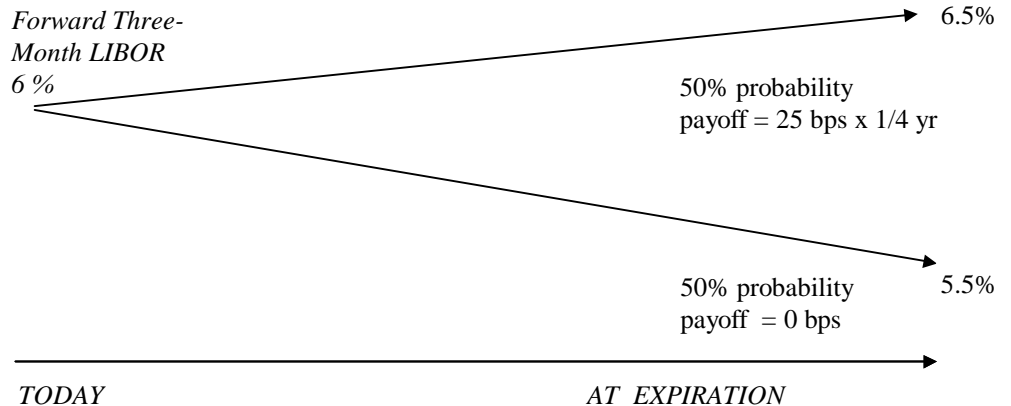
A SIMPLIFIED LOOK AT OPTION PRICING

Interest rate hedging tools such as caps and floors can be priced using the Black option pricing model. The model calculates theoretical option prices using the option's strike, the forward interest rate, the option's time to expiration, implied volatility, and the short-term interest rate.

We can use a simplified option pricing model to illustrate the role of each variable. Below is a simple single-period interest rate cap to show how this interaction works. It is a 6.25% cap on three-month LIBOR six months from now.

CAP THREE-MONTH LIBOR SIX MONTHS FROM NOW CAP STRIKE: 6.25%

$$\begin{aligned} \text{Cap Premium} &= 3.0 \text{ bps} \\ &= \frac{1/2 \times (25 \text{ bps} \times 1/4 \text{ yr}) + 1/2 \times (0 \text{ bps} \times 1/4 \text{ yr})}{1 + (0.059 \times 3/4 \text{ yr})} \end{aligned}$$



An option's price is the present value of the expected payoffs. To value this cap:

- **Find the distribution of possible future rates at the cap's expiration date six months from now.** To keep things simple, assume that today's three-month rate six months forward is 6%. Also assume that three-month LIBOR in six months can have one of only two values - 6.5% or 5.5% - and that each outcome is equally likely. In our example, the ± 50 bps change in three-month LIBOR around today's forward rate represents market volatility.¹
- **Calculate the pay-off for each possible outcome.** If LIBOR is 6.5%, the cap will be worth 25 bps. Since the rate being capped is three-month LIBOR, the payoff will be 1/4 of this value, or 6.25 bps ($1/4 \times 25$ bps). If LIBOR is 5.5%, the cap will expire worthless.
- **Find the expected value of the cap at expiration.** This is the average of the possible outcomes weighted by their probability, or 3.125 bps ($1/2 \times 6.25$ bps + $1/2 \times 0$ bps).
- **Discount the future expected value to find the value of the cap which is paid for today.** The pay-off on the cap occurs three months after its expiration date, or in this example, nine months from now. If we assume a nine-month discount rate of 5.9%, the present value of the cap becomes three bps (the expected future payoff of 3.125 bps $\times 1/(1 + 0.059 \times 3/4 \text{ yrs})$).

This simple example illustrates the importance of the rate distribution at option expiration. The larger the volatility or the greater the time to expiration, the wider the distribution and the more valuable the option.

USING IMPLIED VOLATILITY

We can use implied volatility along with some basic statistics to learn something about the market's expectation of future rate ranges. Statistically, volatility is the annualized standard deviation of daily logarithmic variability. We will show you how to convert an intangible volatility value into a very tangible rate distribution.

For example, if a one-year option has an implied volatility of 15% and an associated forward rate of 6%, there is about a 68% chance that the forward rate one year from now will be in the range of ± 90 bps from the current level. There is about a 95% chance that it will vary ± 180 bps, and about a 99% chance of ± 270 bps. For these approximations, we multiplied the volatility by the forward rate, then doubled and tripled this value.

¹ In our simple pricing tree example, the rate distribution at expiration (6.5% or 5.5%) is symmetric around the current forward rate of 6%. In practice, the Black model assumes that interest rates are lognormally distributed.

But what if the life of our option is six months? Since volatility is an annualized value, we need to translate volatility into the appropriate trading period. Only then can we use it to find the approximate rate ranges over our option's life. For example, the table below shows how we convert a 15% annual volatility to a 10.61% six-month volatility ($15\% \times \sqrt{0.5}$). And by multiplying by the forward rate of 6%, we find that there is about a 68% chance that rates will vary ± 64 bps (i.e., between 5.36 and 6.64) when the option expires six months from now. The table below shows rate ranges for various time horizons.

USING OPTION IMPLIED VOLATILITY TO FIND MARKET EXPECTATIONS FOR RATE MOVEMENTS²

Trading period volatility = Annualized volatility $\times \sqrt{\text{Time}}$

Rate change = Trading period volatility \times forward rate

Forward rate = 6%;

Annualized volatility = 15%

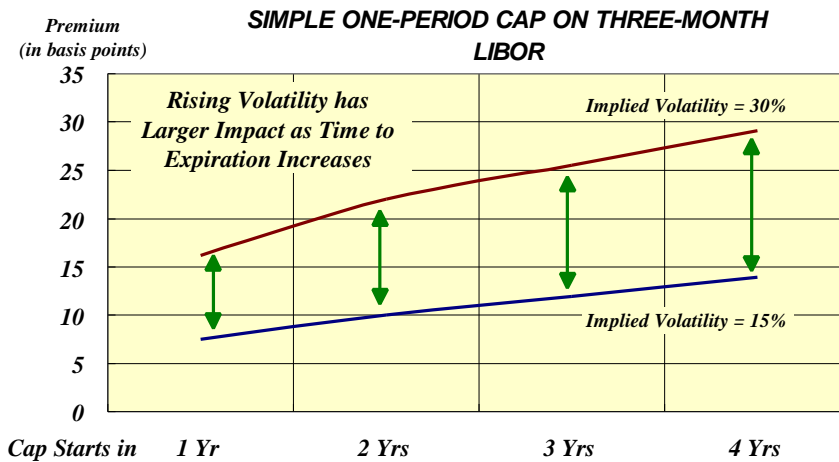
Trading Period	Time	Trading Period Volatility	Rate Change	Low Rate	High Rate
Calendar day	1/365	0.79%	0.05	5.95	6.05
1 week	1/52	2.08%	0.12	5.88	6.12
1 month	1/12	4.33%	0.26	5.74	6.26
1 quarter	1/4	7.50%	0.45	5.55	6.45
6 months	1/2	10.61%	0.64	5.36	6.64
1 year	1	15.00%	0.90	5.10	6.90

² A more precise methodology that incorporates the lognormal distribution of rates uses the formulas

$$Fe^{s\sqrt{\text{Time}}} \text{ and } Fe^{-s\sqrt{\text{Time}}}$$

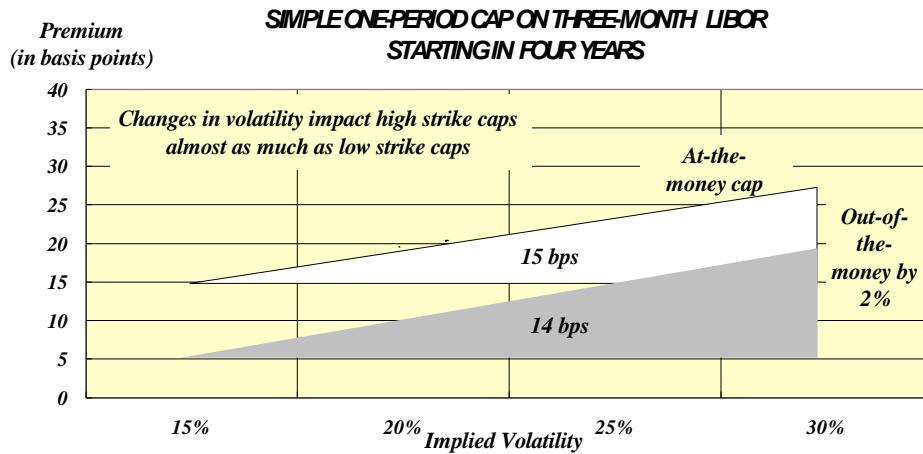
to define the upper and lower one standard deviation bound of rates, where F = forward rate, e is the exponential function, σ is the volatility measure and Time is the option's time to expiration.

IMPLIED VOLATILITY AND HEDGE PRICING



We know that an increase in implied volatility increases the value of an option. But will an increase in implied volatility have the same price impact on short-dated and long-dated options? The answer is no. The chart above tracks how single-period caps with various times to expiration are affected by a doubling of volatility.

Here doubling implied volatility raises the one-year at-the-money cap price by nine bps (from seven to 16). Doubling volatility raises the four-year at-the-money cap price by 15 bps. In absolute terms, the change in implied volatility has a larger effect on the longer-dated option. From a hedger's perspective then, the longer the term of the planned hedge, the more important it is to monitor implied volatility as the execution date approaches.



Will an increase in implied volatility have the same price impact, regardless of the option's strike? It depends. For long-dated options, a change in implied volatility results in nearly the same basis point price change for options whose strikes are at or near the current forward rate (i.e., at or near the money). As the strike strays far from the at-the-money level, the impact of a change in implied volatility lessens. For short-dated options, the strike does not need to move very far from the at-the-money level for this effect to kick in.

In the graph above, with implied volatility at 15%, the four-year at-the-money one period cap is valued at 14 bps, while the 2% out-of-the-money cap is valued at four bps. Doubling volatility to 30% changes the value of both options by almost the same amount (15 and 14 bps, respectively). Note that on a relative basis, this change in implied volatility has a much larger impact on the price of the out-of-the-money cap. Its price increases 350%, while the price of the at-the-money cap doubles. From a hedger's perspective, it is important to understand the interaction between an option's time to expiration and its strike to determine how a hedge will be affected by a change in implied volatility.

SUMMING UP

Understanding what implied volatility tells us about market expectations for future price or rate variability gives us a better way to evaluate our exposure to market risk. Knowing how volatility and changing volatility affect hedge pricing helps us develop cost-effective hedges and appraise the timing risks associated with a particular strategy.

For more information, contact your Bank of Montreal Relationship Manager or one of the Global Financial Product Specialists at the number below.

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