A Comprehensive Macroeconomic Income Determination Model for an Islamic Economy*

MUHAMMAD HUSSAIN

INTRODUCTION

Several attempts at modelling the income determination process in an Islamic Economy have been made.[1] Almost all of these attempts are related to income determination on the demand side of the economy only. To the best of my knowledge no attempt has so far been made to capture the supply-side effects on income determination. Some early Muslim thinkers like Ibn Khaldun (1980) have emphatically stressed the supply-side effects on income determination in a Muslim economy. The main objective of this study is to make an effort to present a comprehensive macroeconomic income determination model for an Islamic economy that takes care of the demand side as far as the goods markets and money markets are concerned and it also covers the supply side of the economy as far as the optimal conditions for hiring of the labour and capital in the factor markets are concerned.

On the demand side the IS – LM model, with amendments as required by imperatives of an Islamic economy, is used. On the supply side a new curve, called YQ curve, representing aggregate supply in r, y space, has been derived from the aggregate production function of the economy and the long-run equilibrium con-

---

[1]In this regard the following writings can be cited:
- (a) Ahmad, Ausaf (1987) *Income Determination in an Islamic Economy*. Jeddah, Saudi Arabia. Scientific Publishing Centre, King Abdulaziz University.
- (b) Sattar, Zaidi (1989) Interest-free Economics and the Islamic Macroeconomic System. *Pakistan Economic and Social Review* 27:2 (Winter) 109–138.
- (c) Tahir, Sayyid, (1987) Macroeconomic Modelling and Income Determination in an Islamic Framework. A paper presented in International Seminar on Islamic Economics for University Teachers, held in Islamabad. Pakistan, under the auspices of Islamic Research and Training Institute of Islamic Development Bank, Jeddah and International Institute of Islamic Economics, Islamabad, from August 22 to September 3, 1987.

ditions in markets of such factors of production as labour and capital. Consequently the model presented in this study becomes a long-run model rather than being a short-run one. Most demand-side models of income determination are considered to be short run models. The present writer feels that the so-called short-run models, which deal with the equilibrium of the investment and savings in the IS curve, no more remain short-run in nature as changes in investment can only take place in the long-run. At the most such models can be called static or comparative static ones as opposed to the dynamic ones.

ASSUMPTIONS

Assumptions of the model presented in this study are:

1. A closed economy is assumed, where the private sector, represented by households and business firms, plays a predominant role. The government plays its regulatory and administrative functions and it is equipped with taxing and spending powers as far as they are allowed by *Shariah*.

2. As interest is prohibited by the injunctions of Islam, therefore interest as a mode of financing is absent in our Islamic economy. It is assumed that *Mudarabah*[2] is used as a substitute for interest. The real rate of profit, which an investor is to share with a financier–*Rabb-al-Mal* (usually a commercial bank), is a factor price for obtaining money capital. In other words we shall consider it as a variable representing cost for obtaining funds for investment in an Islamic economy. Similarly in the money market the real rate of profit obtainable to a financier shall represent the opportunity cost of holding idle money.

3. *Zakah* in accordance with the guidance of *Shariah* is introduced in the economy. It is also assumed that in addition to *zakah* the rich people are in the habit of spending for the cause of Allah.

DEMAND SIDE OF THE MODEL

The demand side deals with the equilibrium in the goods as well as money markets.

In the goods market it is assumed like Ahmad (1987) that consumption of the *zakah* payers will be dependent on their disposable income, which is net of

---

[2]*Mudarabah* is an arrangement whereby funds are provided by one party and the labour (human capital) by another party and the profit of the enterprise is shared by both parties in accordance with agreed upon ratios. In the event of loss entire loss is borne by the provider of funds, while the other party loses its time and effort put in the enterprise.

zakah and other spending made by them for the cause of Allah. Only sources of income for *zakah* recipients are *zakah* and amounts doled out for the cause of Allah by the rich people. They spend all of these amounts on their consumption. Government also charges taxes other than *zakah* in accordance with the guidance provided by *Shariah* as is explained in Hussain (1987, 1993).

The equilibrium in the goods market of the economy requires that aggregate supply of goods (y) be equal to aggregate spending (demand) for goods i.e.

$$y = c + i + g$$

where $\quad c = c_1 + c_2$ $\qquad\qquad\qquad z = z(y)$

$\qquad\qquad c_1 = c\,[\,y - t\,(y) - z - e\,]$ $\qquad\quad e = e(y)$

$\qquad\qquad c_2 = z + e$ $\qquad\qquad\qquad\quad i = i\,(r)$

$\qquad\qquad t = t\,(y)$ $\qquad\qquad\qquad\quad\ g - \bar{g}$

Where all small letters indicate the real values of the variables and y, c, i, g, t, z, e and r represent national income, consumption, investment, government spending on goods and services, taxes, total *zakah* payments or receipts, total spending for the cause of Allah *(infaq fi sabil Allah)* in addition to *zakah* and real rate of profit respectively. c_1 and c_2 represent consumption of the *zakah* payers and *zakah* recipients respectively. $z(y)$, $e(y)$, $i(r)$ are *zakah*, *infaq* and investment functions. The former two are assumed to be functions of income and investment is assumed to be function of the real rate of profit, which investors are to share with the suppliers of funds for investment. All government spending (g) is assumed to be autonomous. The bar on g indicates the autonomous nature of this spending.

The real rate of return or real rate of profit (r) as a factor price for obtaining funds for investment is used in our investment demand function $i = i(r)$ for an Islamic economy rather than using the interest rate as is done in the case of a secular economy. In other words, the real rate of profit (r) is used as a variable representing the cost of obtaining funds for investment in an Islamic economy. This is the approach, which was adopted by Khan (1985) and Haque and Mirakhor (1986). Haque and Mirakhor define the profit rate or rate of return per unit of investment (r) as $r = \dfrac{\lambda\,\pi^{*}}{I^{*}}$, where I^{*} is the optimal level of investment in an enterprise, π^{*} is the optimal profit obtained from that level of investment and λ is the profit sharing ratio. Their conclusion is that in the presence of certainty and competitive conditions, the investment demand schedule will be downward sloping

in r in an interest free Islamic economy. It may be pointed out that with the assumptions of the presence of certainty and competitive conditions *ex post* concept of profit rate (r) is turned into an *ex ante* concept.

Thus IS curve in an Islamic economy will be represented by the following equation:

$$y = c\,[\,y - t\,(y) - z\,(y) - e\,(y)\,] + z\,(y) + e\,(y) + i\,(r) + g \quad \text{...} \quad \text{...} \quad \text{...} \quad (1)$$

The differential of the above IS equation is given by the following expression. It is assumed here that g does not change.

$$dy = c'\,(dy - t'dy - z'dy - e'dy] + z'dy + e'dy + i'dr$$

Where c' is marginal propensity to consume and primes on other letters also indicate the slopes of the respective functions

$$dy - c'\,dy + c't'dy + c'z'dy + c'e'dy - z'dy - e'dy = i'dr$$

$$dy\,(1 - c' + c't' + c'z' + c'e' - z' - e') = i'dr$$

$$\frac{dr}{dy}\Big|_{IS} = \frac{[1 - c'\,(1 - t') - (1 - c')\,(z' + e')]}{i'} < 0 \quad \text{...} \quad \text{...} \quad (2)$$

Where it is assumed that $0 < c' < 1$, $0 < t' < 1$, $0 < z' < 1$, $0 < e' < 1$ and $0 < (c' + t' + z' + e') < 1$.

Keeping in view the assumed values of different parameters, the IS curve in an Islamic economy will be negatively sloped. However, due to presence of z' and e' the value of the goods market multiplier for an Islamic economy will be higher as compared to such a multiplier for a secular economy. Disregarding the Keynesian-Monetarist controversy over the steepness or flatness of the IS curve, the higher magnitude of the goods market multiplier as obtainable in an Islamic economy, would make an Islamic economy's IS curve relatively flatter as compared to such a curve for a secular economy.

In the money market the following equation will represent the LM curve:

$$\frac{\overline{M}}{P} = m = l(r) + k(y) \qquad \text{where} \quad l' < 0 \quad \text{and} \quad k' > 0 \quad \text{...} \quad \text{...} \quad (3)$$

Where $\overline{M}$ represents the exogenously given nominal money supply, P is the general price level and m represents the real money supply. On the right hand side

of the equation, $l\,(r)$ represents demand function for idle money. It is assumed in this function that the real profit rate, which could be obtained when funds are made available for investment on *Mudarabah* basis, represents opportunity cost of holding idle money. In this way demand for idle money becomes a decreasing function of the real rate of profit (r). The transactions demand for money is represented by the function $k(y)$ i.e. this kind of demand for money is dependent on income and it is an increasing function of income.

Differentiation of the LM curve Equation (3) is given below:

$$0 = l'dr + k'dy \qquad \text{where} \quad l' < 0 \quad \text{and} \quad K' > 0$$

The differential of $\dfrac{\overline{M}}{P}$ is zero because M is assumed to be exogenously given and is assumed to be constant for a given LM curve.

$$\frac{dr}{dy}\Big|_{LM} = -\frac{k'}{l'} > 0 \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (4)$$

As $k' > 0$ and $l' < 0$, therefore,

$$\frac{dr}{dy}\Big|_{LM} > 0$$

In other words under our assumptions the LM curve for an Islamic economy will be positively sloped.

SUPPLY SIDE OF THE MODEL

Although the aggregate demand curve is derived by the economists from the IS–LM model, which is represented in r, y space, yet so derived aggregate demand curve is represented in P, Y space. Similarly the aggregate supply curve is also represented in P, Y space. Beck (1979) has pointed out that "use of the conventional aggregate supply curve in y–P space could lead to confusion in this model". Conventionally, the aggregate supply curve is derived from the labour market equilibrium conditions assuming that the capital stock is fixed. Whereas aggregate demand curve in P, Y space is derived from the IS–LM model, which is depicted in r, y space. The IS curve itself entails changes in the capital stock, which contradicts the assumption of fixed capital stock. Consequently equilibrium depicted by the intersection of the aggregate demand and aggregate supply curves in P, Y, space could be inconsistent. In order to remove this inconsistency Beck

has developed a downward sloping YQ curve in r, y space, which represents long-run equilibrium in both capital and labour factor markets simultaneously. Thus YQ curve represents the aggregate supply side of the economy in a more realistic way.

Following Beck's approach, the YQ curve representing the supply side of an Islamic economy can be derived from the conditions for equilibrium in the factor markets of labour and capital by assuming a Cobb-Douglas aggregate production function of the following form:

$$y = N^{\alpha} K^{1-\alpha} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (5)$$

where N and K are labour and capital inputs respectively. We assume that the supply curve of labour, $w(N)$, is upward sloping and workers are not subject to money illusion.

Before a mathematical expression for the YQ curve and its slope are derived, the process of graphical derivation of the YQ curve is explained. In the labour market, the equilibrium level of employment is determined by the demand function for labour represented by the marginal product of labour (MPN) and the supply function of labour, $w(N)$. With a Cobb-Douglas Production Function the marginal product of labour increases when the labour is made to work with a larger stock of capital. Similarly, the marginal product of capital (MPK) increases as the quantity of labour working with capital increases. It is assumed here that factor price for labour is the real wage rate (w) and factor price of capital is the real rate of profit (r), which represents the cost of acquiring investment funds in a profit and loss sharing system.

The YQ curve is graphically derived in Figure 1.

In Figure 1-b, it is shown that marginal product of capital (MPK) curve shifts upward, when the employment level increases from N_1 to N_2. With r_1 and N_1 the equilibrium capital stock is K_1 and with r_2 and N_2 the equilibrium stock of capital is K_2. In Figure 1-c, the marginal product of labour curve is MPN(K_1) with capital stock at K_1 and MPN(K_2) with capital stock at K_2. In the labour market equilibrium occurs at w_1 and N_1 when MPN(K_1) is the demand curve for labour and at w_2 and N_2 when MPN(K_2) is the demand curve for labour.

In Figure 1-d the aggregate production function is $y(N, \overline{K}_1)$, when K_1 is the capital stock, which the labour is to work with. The aggregate production function is $y(N, \overline{K}_2)$, when K_2 is the capital stock, which the labour is to work with. Income level is y_1 with employment level at N_1 and y_2 with employment level at N_2.

In Figure 1-a, by showing supply of output y_1 at r_1 and supply of output y_2 at r_2, YQ curve is obtained in r, y space. This curve represents the aggregate supply
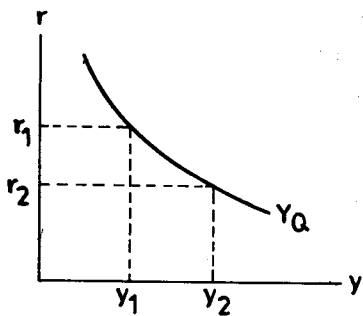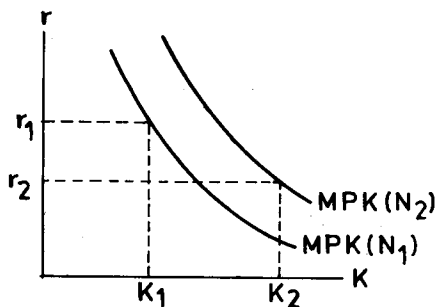
Figure 1

Fig.1-a

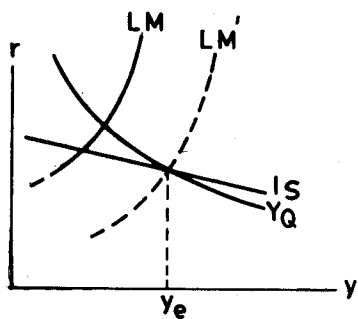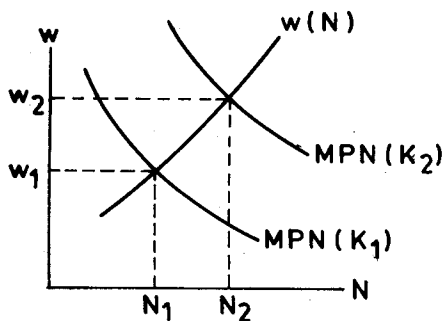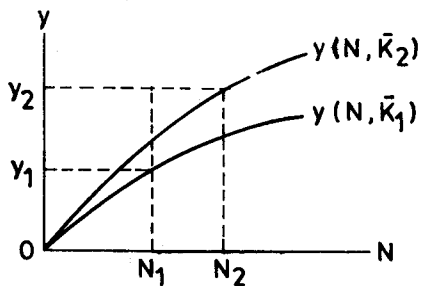## Fig.1-b

Fig.1-e

Fig.1-c

Fig.1-d

of output in an Islamic economy in r, y space while intersection of IS and LM curves represents aggregate demand for output in the same r, y space.

The YQ curve will be downward sloping when the supply curve of labour is vertical or upward sloping. When the supply curve of labour is perfectly elastic, the YQ curve will be a horizontal line. For the sake of ensuring stability of equilibrium of demand and supply sides of the economy, it is assumed that the YQ curve is steeper than the IS curve.

In Figure 1-e IS and LM curves are added to the YQ curve of Figure 1-a. If all the three curves had intersected at a single point that point would have represented the equilibrium of aggregate demand and aggregate supply at the given price level. But in our Figure 1-e the YQ curve intersects the IS curve to the right of the intersection point of the IS and LM curves. This shows that at the given price level aggregate supply is greater than the aggregate demand. Consequently the price level will decline and the LM curve will shift to the right until all the three curves intersect at a single point, which would represent the equilibrium level of output in the economy. In Figure 1-e equilibrium occurs at income level y_e.

Before we discuss the multiplier effects of shifts in either IS or LM curve, we mathematically derive the equation for the YQ curve and its slope.

From the Cobb-Douglas production function given in Equation (5) the equation for the YQ curve can be derived with the help of equilibrium conditions in the factor markets for labour and capital. The equilibrium condition in the case of the factor market for capital is that the marginal product of capital ($MPK = \frac{\partial y}{\partial K}$) is equated to the real rate of profit (r). Because r represents the cost of acquiring investment funds in a profit and loss sharing system:

$$\frac{\partial y}{\partial K} = \frac{(1-\alpha)y}{K} = r \qquad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \qquad (6)$$

Assuming an upward sloping supply curve of labour and that the workers are not subject to money illusion, the equilibrium in the labour market requires that marginal product of labour be equal to the supply function of labour $w(N)$, which represents real supply price of labour.

$$\frac{\partial y}{\partial N} = \frac{\alpha y}{N} = w(N) \qquad \text{where} \quad w' > 0 \qquad \dots \quad \dots \quad \dots \qquad (7)$$

It may be pointed out that four equilibrium conditions in the form of Equations (1), (3), (6) and (7) along with production function Equation (5) gives us five basic equations of this simultaneous equations model to determine values of five unknowns: y, N, K, r and P.

From (6)

$$K = \frac{(1-\alpha)\,y}{r} \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (8)$$

By substituting this value in (5) and solving for N, we obtain the following expression:

$$K = \left(\frac{1-\alpha}{r}\right)^{\frac{\alpha-1}{\alpha}} y \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (9)$$

By substituting (9) into (7) we get the following implicit equation for the YQ curve:

$$y\left[\alpha - \left(\frac{1-\alpha}{r}\right)^{\frac{\alpha-1}{\alpha}} w(N)\right] = 0 \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (10)$$

Differentiation of (7) and (9) results in the following expression for the slope of the YQ curve:

$$\frac{dr}{dy}\bigg|_{YQ} = \frac{-\alpha\,r}{(1-\alpha)\,y}\left[\frac{N^2 \cdot w'}{\alpha y + N^2 \cdot w'}\right] \qquad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \qquad (13)$$

The process of derivation of (13) is given in the Appendix.

The YQ curve will be negatively sloped if the supply curve of labour is positively sloped i.e. if $w' > 0$. If the supply curve is perfectly inelastic i.e. if w' approaches $+\infty$ even then the YQ curve will be negatively sloped and its sloped tends to approach $\frac{-\alpha\,r}{(1-\alpha)\,y}$. If the supply curve were perfectly elastic i.e. if $w' = 0$ then the YQ curve will be horizontal. For ensuring stability it is assumed in our model that the YQ curve is steeper than the IS curve.

MULTIPLIER EFFECTS OF SHIFTS IN EITHER OF THE CURVES CONSTITUTING AGGREGATE DEMAND

Economists usually derive multipliers for shifts in the IS curve and these multipliers are usually confined to the goods market. Values of these multipliers change when the money market is brought into the picture. Values of these multipliers will further change when the supply side represented by the YQ curve is added to the IS–LM demand side model.

Our next task is to graphically show the overall multiplier effects of shifts in either the IS or LM curve, when the supply side is explicitly taken into account.

For example when the IS curve shifts upward as a result of an increase in investment or an increase in government purchases of goods and services or a decrease in the marginal tax rate (t'), then there will be repercussions on both demand and supply sides and the overall multiplier effect is shown in Figure 2.

It may be pointed out that when either government purchases of goods and services increase or when the marginal tax rate is reduced, then the impact effect of these changes will start in Figure 2-b, where initially MPN with the same stock of capital will shift upward. In case of an increase in investment the impact effect of the change will initially start in Figure 2-c, where MPK at the same employment level (N) will shift upward.

Figure 2 shows the ultimate effect of an increase in government purchases of goods and services or a tax rate cut or an increase in investment after taking into account all the repercussions on the demand and supply sides. the aggregate production function in Figure 2-a shifts upward from y $(N, \overline{K}_1)$ to $y(N, \overline{K}_3)$ and in Figure 2-b, the marginal product curve of labour shifts up from $MPN(K_1)$ to $MPN(K_3)$ and in Figure 2-c the marginal product curve of capital shifts up from $MPK(N_1)$ to $MPK(N_3)$ and in Figure 2-d YQ curve shifts up to YQ' and in Figure 2-e on the demand side the IS curve shifts from IS to IS' as a result of either an increase in investment or an increase in government purchases or as a result of a tax cut. In Figure 2-e shift of YQ curve from YQ to YQ' is also depicted. In consequence of these shifts in the curves the aggregate supply becomes larger than the aggregate demand. This will pull the price level down until the LM curve shifts to LM' and IS', YQ' and LM' all intersect each other at one point. The overall multiplier effect of an increase in investment or an increase in government purchases or a tax cut will by $y_1 - y_3$, while if only the demand side were in the picture, the multiplier effect would have remained confined to $y_1 - y_2$.

If an increase in money supply takes place, then cost of obtaining funds for investment i.e. the real rate of profit in a profit and loss sharing system will decrease. This situation will encourage capital formation on the supply side. The effect of this change can be explained with the help of Figure 3. In this case initial impact effect of increase in money supply will start in Figure 3-c where Marginal Physical Product of Capital (MPK) at the same employment level will shift upward as a result of a decrease in the real rate of profit and later on it will have repercussions on all the parts of Figure 3. Figure 3 shows the ultimate effect of an increase in money supply after taking into account all the repercussions on the demand and

supply sides, which are depicted by shifts in the curves in different parts of the Figure. Ultimately to $y(N, \overline{K}_1)$ shifts to $y(N, \overline{K}_3)$ in Figure 3-a and $MPN(K_1)$ shifts to $MPN(K_3)$ in Figure 3-b and $MPK(N_1)$ to $MPK(N_3)$ in Figure 3-c and YQ curve shifts to YQ' in Figure 3-d. In Figure 3-e LM' and YQ' and IS curves intersect each other at income level y_1. The overall multiplier effect of this change is $y_1 - y_3$.
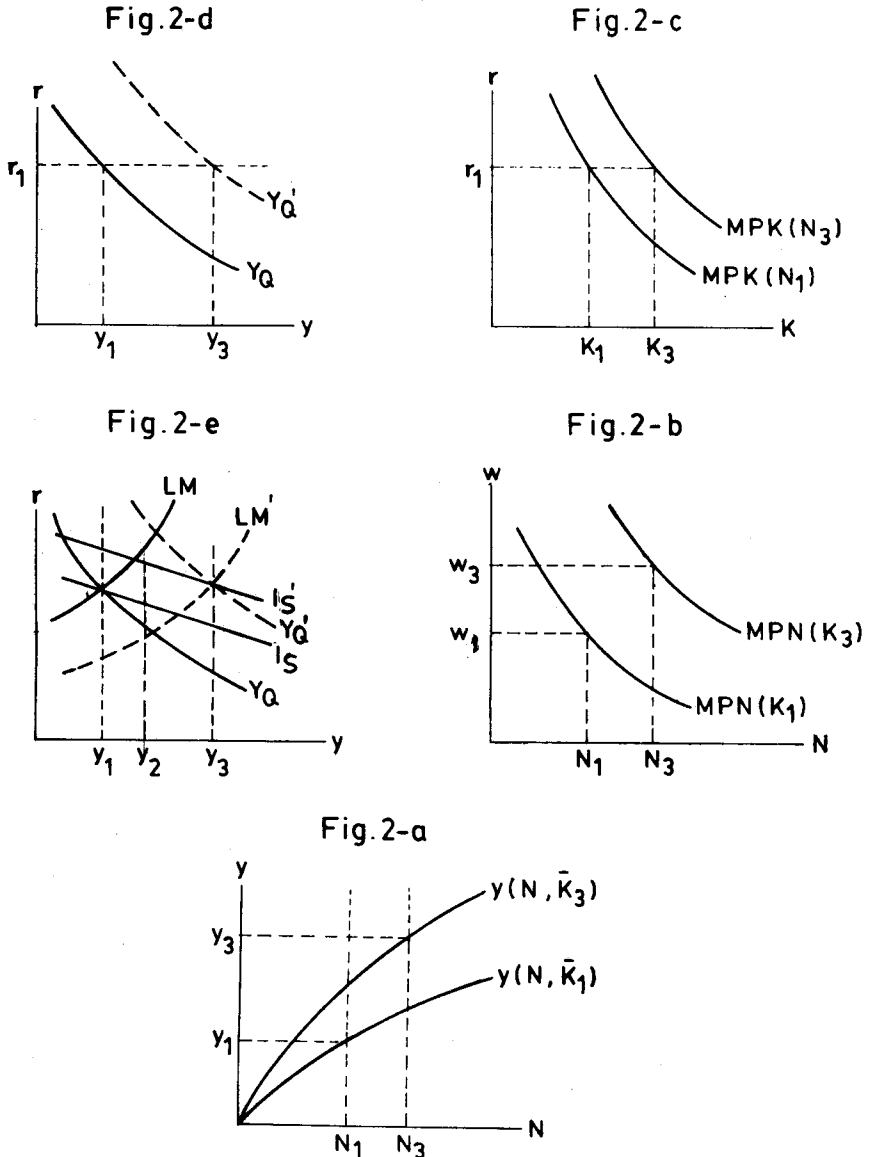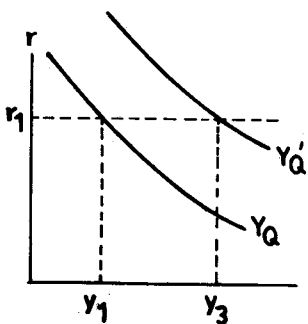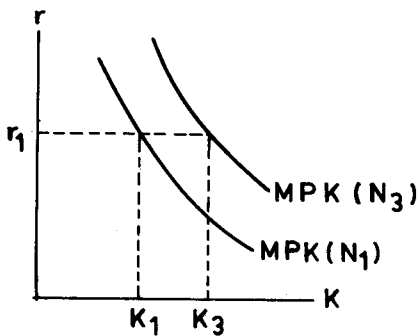
Figure 2

Fig.2-d

Fig.2-c

Fig.2-e

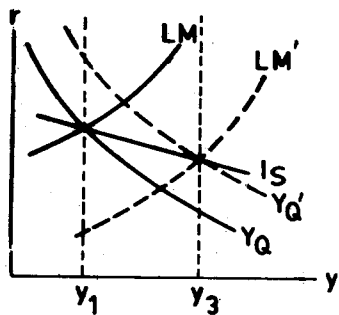Fig.2-b

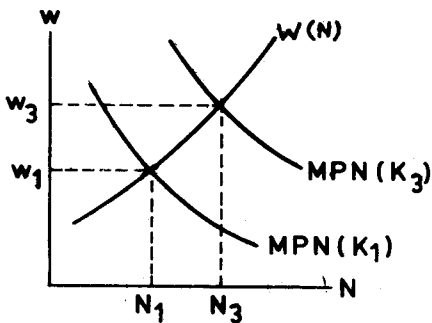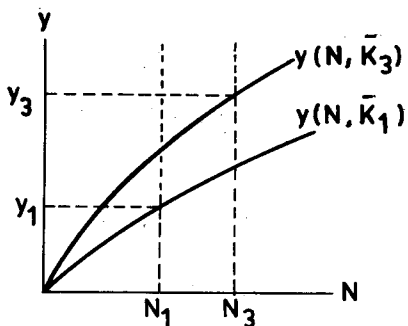Fig.2-a

Figure 3

Fig. 3-d

Fig. 3-c

Fig. 3-e

Fig. 3-b

Fig. 3-a

APPENDIX

The Slope of the YQ curve can be derived by differentiation of Equation (7) and Equation (9). Equation (7) can also be written as

$$y = \frac{N \cdot w(N)}{\alpha}$$

The differentiation of this Equation gives:

$$dy = \frac{w(N) + N \cdot w'}{\alpha} dN \qquad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (11)$$

The differentiation of Equation (9) is given by

$$dN = \frac{N}{y} dy + \frac{1-\alpha}{\alpha} \frac{N}{r} dr \qquad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad (12)$$

as $\quad \dfrac{N}{y} = \dfrac{(1-\alpha)^{\frac{\alpha-1}{\alpha}}}{r}$

Substituting the value of *dN* from (12) in (11) gives

$$dy = \frac{w(N) + N.w'}{\alpha} \left[\frac{N}{y} dy + \frac{1-\alpha}{\alpha} \frac{N}{r} dr \right]$$

$$dy - \frac{N[w(N) + N.w']}{\alpha y} dy = \frac{1-\alpha}{\alpha^2} \frac{N}{r} [w(N) + N.w'] dr$$

$$dy \left(1 - \frac{[w(N) + N.w']N}{\alpha y} \right) = \frac{1-\alpha}{\alpha^2} \frac{N}{r} [w(N) + N.w'] dr$$

$$\frac{dr}{dy}\bigg|_{YQ} = [1 - \frac{[w(N) + N.w']N}{\alpha y}] \left[\frac{1}{\frac{1-\alpha}{\alpha^2} \frac{N}{r} [w(N) + N.w']} \right]$$

$$= \frac{\frac{\alpha^2 r}{1-\alpha} \alpha y - [w(N) + N.w'] N \frac{\alpha^2 r}{1-\alpha}}{\alpha y.N [w(N) + N.w']}$$

$$= \frac{\frac{-\alpha^2 r}{1-\alpha}\left[-\alpha y + N\left[w(N) + N.w'\right]\right]}{\alpha y\left[w(N) + N.w'\right]N}$$

$$= \frac{-\alpha^2 r}{(1-\alpha)\,\alpha y}\left[\frac{-\alpha y + N\left[w(N) + N.w'\right]}{[w(N) + N.w']N}\right]$$

Substituting the value of $w(N)$ from Equation (7) :

$$\frac{dr}{dy}\Big|_{Y\varrho} = \frac{-\alpha r}{(1-\alpha)\,y}\left[\frac{-\alpha y + \alpha y + N^2.w'}{\alpha y + N^2.w'}\right] = \frac{-\alpha r}{(1-\alpha)\,y}\left[\frac{N^2.w'}{\alpha y + N^2.w'}\right] \quad \dots (13)$$

REFERENCES

Ahmad, Ausaf (1987) *Income Determination in an Islamic Economy*. Jeddah: Saudi Arabia. Scientific Publishing Centre, King Abdulaziz University.

Beck, John H. (1979) An Analysis of the Supply-side Effects of Tax Cuts in an IS-LM Model. *National Tax Journal* (December) 32: 493–499.

Haque, Nadeem ul, and Abbas Murakhor (1986) Optimal Profit-sharing Contracts and Investment in an Interest-free Islamic Economy. Mohsin S. Khan and Abbas Mirakhor (eds) *Theoretical Studies in Islamic Banking and Finance*. Houston, Texas: The Institute of Research and Islamic Studies.

Hussain, Muhammad (1987) Rationale of Taxation in an Islamic Framework and Islamic Principles of Taxation. Presented in the Seminar on State Finance in Early Islam held at Yarmouk University, Irbid, Jordan, 5–8 April.

Hussain, Muhammad (1993) A Macroeconomic Taxation Model for an Islamic Economy. *The Pakistan Development Review* 32:4 947–958.

Ibn Khaldun (1980) *The Muqaddimah* 2. Translated from Arabic by Franz Rosenthal. Princeton: New Jersey Princeton University Press.

Khan, Mohsin S. (1986) Islamic Interest-free Banking: A Theoretical Analysis. *IMF Staff Papers* 33:1 (March) 1–27.