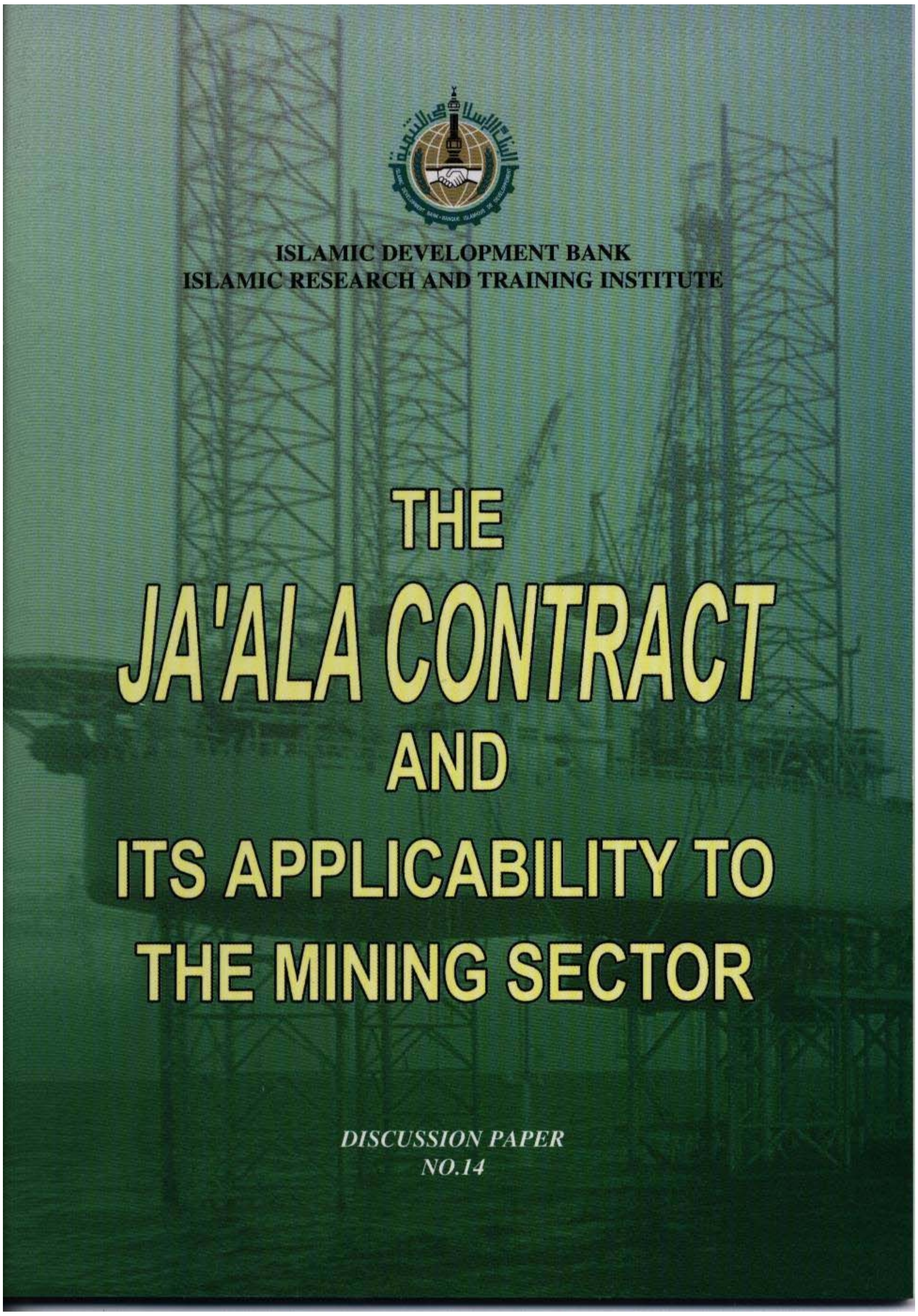
ISLAMIC DEVELOPMENT BANK
ISLAMIC RESEARCH AND TRAINING INSTITUTE
THE
JA'ALA CONTRACT
AND
ITS APPLICABILITY TO
THE MINING SECTOR
DISCUSSION PAPER
NO.14

ISLAMIC DEVELOPMENT BANK
ISLAMIC RESEARCH AND TRAINING INSTITUTE

THE *JA'ALA* CONTRACT AND ITS APPLICABILITY TO THE MINING SECTOR

By
Dr. Boualem Bendjilal ***DISCUSSION PAPER NO.14***

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In the Name of Allah, the Most Merciful, Most Beneficent

Contents

FOREWORD

The Islamic Research and Training Institute (IRTI) of the Islamic Development Bank (IDB) has been established in 1401H (1981) "to undertake research for enabling the economic, financial and banking activities in Muslim countries to conform to *Shari'ahh*". This is in implementation of the article (2) of the Articles of the Agreement establishing the bank, which has been ratified by 54 Islamic countries. In order to discharge its responsibilities, IRTI pays special attention to basic and applied research in the areas of Islamic economics, banking and finance and economic cooperation among OIC member countries.

IRTI researchers carry out research activities on various important issues of Islamic economics, Islamic banking and finance and economic cooperation and development. The Institute also encourages and promotes research activities by outside scholars. It invites eminent economists and *Shari'ah* scholars to deliver lectures on various issues of interest. It organizes research seminars, workshops and training courses. It also awards an international prize that alternates between Islamic economics and Islamic banking every year.

The present research on "*The Ja'ala Contract and its Applicability to the Mining Sector* " is undertaken as part of IRTI's annual research program .This study constitutes an embryo that serves to develop in the future a general framework towards formulating an Islamic contract theory with applications to the different sectors of the economy. As a first step, we focus our attention on the *Ja'ala* contract as an alternative for the existing concession contract. The study discusses how the contract may be used to exploit the mining sector.

The study provides an overview of the *Shari'ah* framework that governs the *Ja'ala* contract. It lays down the main features of this contract and spells out explicitly one of its main features that says "*The remuneration may be agreed upon between the Jae'el and the Maj'ul to be part of the capital lost or a fraction of the output"*. The paper gives a full characterization of the *Ja'ala* contract in both cases in discrete time as well as continuous time. It derives the bilateral relationship between the *Jae'el* and the *Maj'uul* and shows how the cost efficiency parameter of the *Maj'uul* and the extraction rate of the mineral interact in general .

In addition, the study investigates in detail the symmetric information case. It derives explicitly the optimality conditions and the trajectory of the extraction rate of the mineral. It shows that the extraction rate of the mineral depends on several parameters namely the cost efficiency parameter of the *Maju'ul,* the social planner's time preference rate, the market price of the mineral and the geological nature.

Lastly, the study gives some indications on how this research can be extended to include the exploration-extraction case and the asymmetric information case as well as on the possibility of relaxing the assumption of the exogenous property of the price of the mineral.

It is hoped that the publication of this monograph would encourage and lead to similar efforts so that future developments in this area induce more empirical investigation and discussion.

BASHIR ALI KHALAT
Acting Director, IRTI

ABSTRACT OF THE *JA'ALA* CONTRACT

This study constitutes an embryo that serves to develop in the future a general framework towards formulating an Islamic contract theory with applications to the different sectors of the economy. As a first step, we focus our attention on the *Ja'ala* contract as an alternative for the existing concession contract. The study discusses how the contract may be used to exploit the mining sector.

First, the study provides an overview of the *Shari'ah* framework that governs the *Ja'ala* contract. It lays down the main features of this contract and spells out explicitly one of its main features that says "*The remuneration may be agreed upon between the Jae'el and the Maj'ul to be part of the capital lost or a fraction of the output*". This feature serves as one of the key assumptions in this research. A comparison between the main features of the *Ja'ala* contract and other Islamic modes of finance is also given.

Second, the paper gives a full characterization of the *Ja'ala* contract in both cases in discrete time as well as continuous time. It derives the bilateral relationship between the *Ja'eel* and the Maj'ul and shows how the cost efficiency parameter of the *Maj'uul* and the extraction rate of the mineral interact in general. Moreover, the study shows how the trajectory of the extraction rate of the mineral over the agreed period is determined based on the information provided in the report of the *Maj'uul*.

Third, the paper studies in detail the symmetric information case. It derives explicitly the optimality conditions and the trajectory of the extraction rate of the mineral. It shows that the extraction rate of the mineral depends on several parameters namely the cost efficiency parameter of the *Maj'uul*, the social planner's time preference rate, the market price of the mineral and the geological nature. It shows that the extraction rate of the mineral is inversely related to the preference of the *Ja'eel*, but positively related to the market price of the mineral. Moreover the study derives the cost efficiency parameter of the *Maj'uul* and shows

that it is positively related to the price of the mineral and inversely related to the difference between the initial stock and the stock left over as well as to the *Ja'eel*'s time preference rate.

Finally, the study derives a critical value of the efficiency cost parameter of the *Maj'uul* corresponding to the case where the left over of the mineral after the agreed period is no more profitable. Lastly, the study gives some indications on how this research can be extended to include the exploration-extraction case and the asymmetric information case as well as on the possibility of relaxing the assumption of the exogenous property of the price of the mineral.

1
INTRODUCTION

1.1 Importance of the Mining Sector

Mining industry is considered one of the vital industries that a country needs to develop its own economy. This is the sector, which produces minerals and materials that cater to developmental needs in a country. On the national level it will have a significant macroeconomic and fiscal impact. It plays a vital role in contributing to the improvement of budgetary resources necessary for the economy. The importance of this sector therefore, requires its optimal management in order to generate sufficient revenues, which can be used as an engine to enhance the overall economic growth and to finance the social sector. Moreover, they may play a positive role in poverty reduction programs. Large mining operations invest substantially in local economic development in terms of training, social services and public goods like transport, energy, and infrastructure.

The private sector also has a lot to expect from mineral extraction. World Bank's estimates show that the approximate share of the mining sector ranges between 5% and 10% for fiscal revenue; 15%-25% of export earnings and 3% - 5% of GDP. It constitutes between 10% and 15% of the industrial workforce[1]. We may conclude from these estimates that this sector is very crucial for an economy.

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[1] Substantial fiscal impact from mining, contributing to economic and social development, can be found in several countries. Tax receipts from a single mining company can amount to 30% to 50% of a country's fiscal income such as in Botswana and in Guinea small-scale mining also provides employment for about 13 million workers and their families in particular in countries such as Burkina Faso, Congo, Ghana, India, Indonesia, Brazil , Bolivia and Thailand among others. On the other hand, large-scale mining provides direct employment for about 2-3 million workers and their families worldwide. For every job created directly in the mining sector, between 2 and 25 jobs are created with suppliers, vendors and contractors to the mine and to miners and their families. They are typically provided through small and micro-enterprise activity. World Bank African Mining Report 2000 Investment and Business Opportunities (Symposium 12/2000) www.worldbank .org/mining.

Muslim countries are fortunate enough to have vast mineral resources at their disposal, while their people seem not benefiting from this wealth. It is experienced that these countries may be facing difficulties in terms of finance and expertise in exploiting properly these resources to their own advantage . Therefore, they find no other option but to rely on foreign companies for the exploration and extraction of their mineral resources.

Given the significance of the mining sector and its crucial role in boosting the economic development ; Muslim countries look for mining concession contracts to maximize the welfare of their people .

1.2 What is a Concession Contract ?

Several types of concession contracts are found in the literature. The most often used concession contract is Build, Operate and Transfer (B.O.T.).[2] In a B.O.T. concession contract, the government delegates the rights to a promoter for building a public infrastructure using his own financial means. In return, the promoter will have the right to reap the benefits of the so built public infrastructure for a certain agreed period ranging between 10 to 30 years. The accrued benefits may depend on the size and type of infrastructure. Having benefited from the infrastructure for the agreed period it is transferred to the government. In general, the promoter adds an arbitrage clause in the concession contract that prevents the government from transferring the license to a tierce or from acting in a non-desirable manner unilaterally[3].

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[2] The other forms of concession contracts include the BOOT (Build, Own ,Operate, and Transfer), the BOO (Build, Own and Operate) and the ROM (Renovate, Operate, and Maintain). Bureau, Volume 30, No. 17, October 15th 1999.http://www.barreau. qc.ca/journal/vol 31/No17/imprevisible.html.

[3] Another clause is usually added to these contracts that distinguishes between natural circumstances and political circumstances in order for the government not to take advantage of the geographical location of the project. The two parties fix also in a concession contract what they call adaptability conditions. These include clauses related to monetary indexation, revision of prices according to the price variation in the market and quantitative adjustment i.e., the quantities should adjust according to the market forces. The seller should accept to decrease the prices of the quantities agreed upon in the contract by the same increment as he does it for his competitors.

In the mining sector, however, an exploration permit is first granted to the promoter by the government for a period of 4 or 5 years on average, depending on the type of mines, the type of existing infrastructure, the geographical location and other factors. The exploration contract is usually renewable for a number of times say, two to three times.[4]

Should the exploration phase be successful, the government will delegate the extraction-operation to a specialized firm. The mining prospector who holds the rights of the discovery may also enter into a contract with a mining firm specialized in exploitation to carry out the exploitation of the mine[5].

The exploitation of non-renewable resources like mines is often organized in the form as bilateral relationship between the owner of the mine and the specialized firm in mining extraction. Nowadays, the more often used framework in practice takes the following form. The state delegates the extraction of the mine to a specialized firm. In return, the firm pays to the government periodical payments called royalties.

1.3 Literature Survey, Objective and Organization of the Paper

For the past several years, the economic analysis on mining concession has focused on the bilateral relationship between the owner of the mine and the agent or the firm specialized in mining exploitation. This approach raises the question of the determination of an optimal concession contract, which has to take into consideration the private

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The government should commit himself to grant the same benefits to the actual concessionaire as he does it for the ulterior ones.

4 During the exploration phase, the permit holder is exempt from the sales tax and duties on imported equipment and supplies necessary for exploration activities, as well as on fuel used for operation of fixed installations.

5 The mining permit is in general granted for a fixed period (five years on the average for most of the cases) and is renewable while a mining concession is valid for a longer period which may go up to 25 years and which is renewable too. During the construction and commissioning period, the specialized firm holding the permit benefits from a special tax regime aimed at the promoting of the development of the mining sector in the original country. It is also customary that the mining company which operates autonomously in its activities is allowed to expatriate its profits.

information on the extraction costs, which is known only to the specialized firm. This asymmetry of information creates a situation where adverse selection may occur and therefore, the optimal royalty must reckon information constraints. These constraints are discussed by Karp and Livernois (1992) Gaudet and Al (1991,1nd 1995) and Poudou and Al. (2001).

The economic analysis on mining concession constitutes an application of the well-known principal–agent problem with adverse selection, which is extensively studied and applied to various situations with asymmetry of information. A detailed discussion on the subject may be found in Baron and Myerson (1982); Guesnerie and Laffont (1984) and Laffont and Tirole (1986). The problem of asymmetric information arises with respect to the size of the stock of the mine and its true cost of extraction which is fully known only to the specialized firm. The owner of the mine knows only what the firm reports to him[6].

Islamic contracts have been extensively studied by many Muslim scholars[7]. However, at least to the best of our knowledge only few studies tried to analyze the *Ja'ala* contract. Among these studies we have Dunia (1994), Fahim M. Khan (!995) and Al-Suwailem S. (2000). Dunia studied the *Ja'ala* contract from the *Fiqhi* point of view. He analyses the opinions of the different *Fuqaha* on the subject and in particular the opinions related to the four schools of Islamic jurisprudence . He concluded that the majority of the *Fuqaha* of these four schools considered the *Ja'ala* contract as licit. He pointed out the main foundations that govern the *Ja'ala* contract. Finally, he mentioned some examples where the contract may be applied.

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6 Osmundser (1998), Salanie (1994), Baron (1981), and Besanko and Sappington (1987) designed a royalty schedule and a rate of extraction of the mine to be proposed to the agent (the firm specialized in extraction). This will induce the latter to reveal the private information it holds about the cost of extraction.

7 A number of authors like Abdul Halim, Umar M. (1995), Chapra, M.Umar (1985), Al-Amin, Hassan (1994), Khan, Fahim (1991), Khaled, Rachid (1979), and Abu Sulaiman Abdu Al Wahab (1992), and Abdul Halim, Umar (1995), have studied Islamic contracts. The list is not exhaustive. Their application to the banking, agriculture, trade, and international trade has been explored by Al-Nimri Bin Saleh (1410H), Bendjilali, B. (1994), Gulaid, Mahmoud (1995) Ahmed, Ausaf (1993), Ausaf and Khan, Tariqullah (1997), Al-Nimri, Khalaf Bin Saleh (1410),El-Hennaoui, Mohammed (1993), and Hamoud, Sami (1987), among others.

In his study on "*Islamic Futures and their Markets*" Khan explored the usability of the *Ja'ala* contract as a base for developing futures markets. The study spelled out the main characteristics of the *Ja'ala* contract and compared it with *Istisna'* and *Ijara* contracts. The author pointed out also some important areas of application without going into details. The study concludes that futures markets on the bases of *Ja'ala* need more careful study and investigation. Al-Suwailem did not elaborate much on the *Ja'ala* contract in his study on "Measure of Gharar in Exchange". The author tries to argue that in contrast to Gharar where there is no room for cooperation according to him, the *Ja'ala* contract viewed as a zero-sum game allows both the *Ja'eel* and the *Maj'uul* to cooperate.

The first study focused on *Fiqhi* oriented aspects with some examples. The second and third studies did not go deep into the analysis of the *Ja'ala* contract per se: Both authors devoted only a section to this contract in their whole studies because the *Ja'ala* contract was not their main focus.

In addition, not much attention has been given to the economic analysis of the *Ja'ala* contract and its application to the different sectors and in particular to the mining sector. Therefore, it calls for more effort to have a rigorous analysis of the contracts in general, and of the *Ja'ala* contract in particular.

The long-term objective of this paper is to develop a general framework for the *Ja'ala* contract towards formulating an Islamic contract theory with application to different sectors of the economy. The idea is as follows for each contract starting with the *Ja'ala* contract. We will try to set up its characterization and lay down the main assumptions that govern this contract. Having derived each characterization and the set of norms governing each contract, we look for a common characterization and a common denominator of set of assumptions. This common characterization and common denominator of set of assumptions will then serve as the core for all Islamic contracts. Based on that common characterization and that minimal set of assumptions a theory of Islamic contracts may then be developed. All known Islamic contracts will share this common characterization and minimal set of assumptions. Each specific contract will have this common characterization and the minimal set of assumptions as a base in addition to a number of other assumptions peculiar to that specific contract. Based on this idea, whenever a new economic situation arises that needs a new contract, one can think of developing a new version taking the common characterization and the minimal set of assumptions as a base to which will be added the new assumptions that fit that particular economic situation. This may be looked as part of financial engineering which constitutes nowadays an important area of research for the development of new financial instruments.

The study constitutes an embryo towards reaching this goal. It focuses on the *Ja'ala* contract as an alternative for the existing concession contract. It studies only in detail the extraction case under symmetric information. Some indications are given in the conclusion for a possible extension of this research to include the asymmetric case as well as the extraction –exploration case.

The paper is organized as follows. Section Two presents an overview of the *Shari'ah* framework of the *Ja'ala* contract. In this section, the definition, the status and a summary of the main features of the *Ja'ala* contract are given which are crucial to the formulation of the basic model. In Section Three, we design the basic model and lay down its characterization, and its assumptions. The model is set up in such a way that it can be used to analyze both cases; the extraction and the exploration-extraction setting. Section Four derives the optimality conditions under the symmetric information case and gives economic interpretation to the results. Section Five concludes the study by giving some indications on how this research can be extended to include the exploration-extraction case, the asymmetric information case and the possibility of relaxing the assumption of the exogenous property of the price of the mineral.

2
THE *SHARI'AH* FRAMEWORK OF THE *JA'ALA* CONTACT

2.1 Definition of the *Ja'ala* Contract

The *Ja'ala* is a contract composed of three elements, the *Ja'eel* (principal), the *Maj'uul (agent)* and the *Ju'ul* (remuneration or compensation) in which a *Ja'eel (p*rincipal) hires a *Maj'uul* (agent) to perform a given task for a given *Ju'ul* (a compensation in monetary terms or otherwise as it is specified in the contract) under the following conditions:

1. If the *Maj'uul* completes successfully the task agreed upon, the *Ja'eel* pays him the agreed *Ju'ul* (compensation or remuneration).
2. If not, the *Maj'uul* gets nothing.

2.2 Status of the *Ja'ala* Contract in Islamic Jurisprudence

Scholars of Islamic jurisprudence are divided on whether the *Ja'ala* contract is permissible in Islamic *Shari'ah*. According to the majority of the Malikite, Shafiite, Hambalite and Shiite scholars, *Ja'ala* is a permissible contract to be adapted in transactions[8]. On the other hand, the majority of the Hanafi and Dhahirri scholars take an opposite view that the *Ja'ala* contract is not compatible with Islamic *Shari'ah*. Hanafi scholars argue that this contract is illicit because it has elements of gambling (Qimar) and risk (Mukhatara) on one hand, while it also deals with an unknown person on the other hand. The Hanafi scholars consider the *Ja'ala* contract as null and void (Batil) if the agent (*Maj'uul*) is not

8 For a more detailed discussion on the subject see Ibn Rushd "Al Mukaddimah" Vol. 2 Dar Saddar Beyrouth or Al-Ramli "Nihayatul Mujtahid" Vol. 5, p 465 ,Library Al Halabi Egypt or Muhammed Jaouad Mughnia "Fikh Al Imam Jaafar Al Sidiq" Vol. 4 p 293 Dar Al Ilm Lilmalayin, Beyrouth.

determined. However, the Dhahiri scholars do not consider the *Ja'ala* as a contract but as a promise which is preferable to be kept[9].

2.3 Main Features of the *Ja'ala* Contract

The main features of this contract may be summarized as follows:

1. The agent may accomplish the task alone or in partnership with others in the framework of this contract. He also may delegate the task to a tierce person to do it for him. The principal pays the agent his remuneration. The agent in his turn will pay as mutually agreed to his partners or the agent he delegated. The agent can also sign a fresh *Ja'ala* contract (a parallel *Ja'ala* contract to the first one) with a new agent to perform the task for a given *Ju'ul*. Here the remuneration of the second agent will be naturally less than that of the first agent; otherwise he will not sign a new *Ja'ala* contract. The first agent will then get a margin from this new *Ja'ala* contract. The first agent in this case plays the role of the principal (*Ja'eel*) while the second agent serves as the *Maj'uul* (agent).

2. In the *Ja'ala* contract sometimes an agent may be specified or sometimes not. In case the agent is not specified at the first hand, the *Ja'eel* (the principal) will pay the compensation as he announces to the person who does the job. However, this kind of situation may not work in the contemporary setup but *Fuqaha* envision a possibility of such a case.

3. The *Ja'eel* (principal) may contract more than one *Maj'uul* (agent) to perform the task .The remuneration will then be shared by the agents as agreed.

4. In the *Ja'ala* contract the nature and amount of remuneration should be specified in clear terms so that there will be no

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9 For a detailed discussion on the subject see Ibn Hazm "Al Mouhalla" Vol. 8 pp.193 and 204. Also, see Khaled Rachid (1979) "Al Jouala Wa Ahqamuha" Baghdad University and "Chaouki Dunia (1994)" *Juaa'la* and *Istisna'*: "Economic and Judicial Analysis", IRTI/IDB.

ambiguity in this respect . It also requires the nature of the task to be well specified.

5. In practice, the remuneration may in general be agreed upon to be part of the capital lost or a fraction of the output[10]. In case of disagreement, the agent will be paid a wage like a paid employee[11]. In the case of mining for instance, the *Maj'uul* may be paid a fraction of the production of the mineral extracted as a remuneration. In the case of debt recovery, the agent or firm specialized in debt recovery may get a percentage of the debt recovered as *Ju'ul*.

2.4. The *Ja'ala* contract versus *Ijara* and *Istisna'* contracts and its contemporary set-up.

The *Ja'ala* contract has several similarities with Ijara and *Istisna'*. In Ijara and *Ja'ala* contracts the object is the same. They render services against a determined remuneration. In both contracts, the remuneration has to be well-defined and known to all parties. Nevertheless, there exist some dissimilarity. The Ijara contract is a bilateral contract between two well-known parties whereas in the *Ja'ala* contract the *Maj'uul* may not be known. The *Ijara* contract is a binding contract (*mulzam*) whereas the *Ja'ala* contract is a permissible contract (*jaeez*). Whereas in *Ijara* contract the worker may get his salary either in installments or even in advance, in *Ja'ala* contract the remuneration must be given only after completion of the task. In *Ja'ala* contract, the two parties may not necessarily be present at the time of signing the contract whereas, in *Ijara* contract the two parties have to be present. Moreover, the method by which the service has to be rendered in the *Ja'ala* contract does not have to be specified whereas in *Ijara* contact all the elements have to be well-specified. In *Ja'ala* contract, the *Ja'eel* may not be the owner whereas in *Ijara* contact this is not possible.

Ja'ala and *Istisna'* contracts have also similar features. However, there exist also some differences. In *Istisna'* contact, the *Sana'* (seller)

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[10] For a detailed discussion on this point see Sahnoun " Al Mudawana " Vol. 5, p.452, Dar Sadar, Beyrouth, Lebanon.

[11] For a discussion on the subject see Al Ramli " Niyat Al Muhtaj", Vol. 5.

provides a physical commodity whereas in *Ja'ala* contract the *Maj'uul* provides a service. In *Ijara* and *Istisna'* contracts, the remuneration of the service is known with certainty whereas in *Ja'ala* contact the payment of the service is tied to the result which is undetermined and uncertain.

The *Ja'ala* contract is useful when time and effort are uncertain. Let us take the example of the mining sector where the good exists but its delivery is not certain due to the uncertainty of the cost of exploitation and other uncertainties of geological nature such as depth, nature of the rocks, etc.

There may also be other areas where *Ja'ala* contract can apply. An important field of application of the *Ja'ala* contract is research and development. A research and development institute for example may be contacted by a state or a firm to invent a new product against a given remuneration. The payment is made at the time of the delivery of the product. In this situation, the delivery itself and the time of delivery of the product are both uncertain. In these specific situations, contracts like *Istisna', Salam* or *Ijara* etc. are not feasible given the existence of uncertainty involved in both the delivery of the product and its price.

The *Ja'ala* contract may be used as a base for financial intermediation. An Islamic bank may function on the basis of the *Ja'ala* contract in two ways. First, the bank may play the role of *Maj'uul* (agent). In this case, the bank as *Maj'uul* (agent) gets remuneration for the services (mainly those that involve uncertainty) it will render to its client who plays the role of *Ja'eel*. Services like collection of debts for instance which involve uncertainty are potential candidates. Therefore, banking services can be rendered on the basis of the *Ja'ala* principle between the client and the bank with specific fees for specific services. This first relationship between the bank and the client is not at the advantage of the bank because the bank may spend a lot of efforts and resources without getting any return. Second, the bank may use the principle of "*AL Ja'eel Yu Ja'eel*" like the case where the bank uses the principle of "Al-*Mudharab Yu Dharib"*. Using the principle of "*Al Ja'eel Yu Ja'eel"*, the bank will contract a new parallel *Ja'ala* contract for a lower *Ju'ul* (low remuneration) with a specialized firm that will perform the task. The bank will play in this case the role of *Ja'eel*. The bank will take the difference between the remuneration of the first *Ja'ala* contract signed between the depositor (client) and the bank and the remuneration of the second *Ja'ala* contract signed between the bank and the specialized firm. By using this principle the bank will be playing pure financial intermediary role.

3
THE BASIC MODEL

3.1 Preliminaries

To model the *Ja'ala* contract with application to the mining sector, we take the following assumptions:

A1. The *Ja'eel* is the owner of the rights to a stock of a mine (resource) S_0 and delegates the extraction of the mine to a risk neutral *Maj'uul* (firm) specialized in mining extraction. For the sake of the discussion, we will think of the *Ja'eel* as being the state throughout the study. The *Ja'eel* is assumed to play the role of the state in the rest of the paper. Consequently, the *Ja'eel* maximizes the expected social welfare function over the agreed time period.

A2. The extraction lasts for a period [0,T]. For simplicity of the exposition, we assume that the mine becomes worthless after time T. It is therefore not optimal to exploit the mine beyond time T.

A3. The *Maj'uul's* costs at period t are given by some function $C(\theta, q_t, S_t)$ where θ, q_t, and S_t represent respectively the efficiency cost parameter, the extraction rate or the quantity of the resource extracted at time t, and the stock of the mine at time t.

A4. The *Ja'ala* contract is based on assumption 5 of Section 2.3, which considers the *Ju'ul* (remuneration) as a fraction of the output. That is the remuneration of the *Maj'uul* is equal to a percentage of the output extracted.

A5. At the time of signing the *Ja'ala* contract the exploration phase of the mine is assumed to be already completed.

A6. The *Ja'eel* can observe the quantity extracted but cannot verify the cost actually incurred by the *Maj'uul*.

A7. The market price of the output p_t is a common knowledge to all parties and is treated as an exogenous variable.

From the above assumptions, we may have two different situations, the symmetric and asymmetric information. The first situation arises when the resource price and the extraction costs are perfectly observable by both the *Ja'eel* and the *Maj'uul*. The second situation arises when in practice only the *Maj'uul* will fully know the true costs of extraction, while the *Ja'eel* may not be aware of it. In a situation of asymmetry of information, the *Ja'eel* can observe the quantity extracted but cannot verify the actual cost of the *Maj'uul*. The *Ja'eel* will expect the *Maj'uul* to hide the true cost of extraction whenever it suits his own interest. Therefore, the *Ja'eel* may face information constraints to determine the optimal level of *Ju'ul* for each period.

There are three essential questions that the *Ja'eel* faces when maximizing the expected social welfare, defined to be the present value of profits from the extraction operation over the whole period [0,T].

1. How much does the *Ja'eel* have to pay the *Maj'uul* to cover the cost of providing the service including some profit in case the latter accepts to accomplish the task?

2. How much is the *Ja'eel* going to allow the *Maj'uul* to extract per period? In other words, what is the extraction rate for each period?

3. How is the *Maj'uul* going to collect the money to cover his expenses and in particular the expenses related to the first period?

The answer to these questions will fully characterize the *Ja'ala* contract. It determines the optimal *Ju'ul* (that is the payment schedule J_t for all t) that will maximize the expected social welfare over the whole period [0,T].Next, we will try to give a complete characterization of the *Ja'ala* contract .

3.2 Set-up of the Model and Characterization of the *Ja'ala* Contract

In this section, the model will be set up in its general form with a complete characterization of the *Ja'ala* contract.

The *Ja'ala* contract may be defined as a bilateral relationship between the *Ja'eel* and the *Maj'uul* as follows:

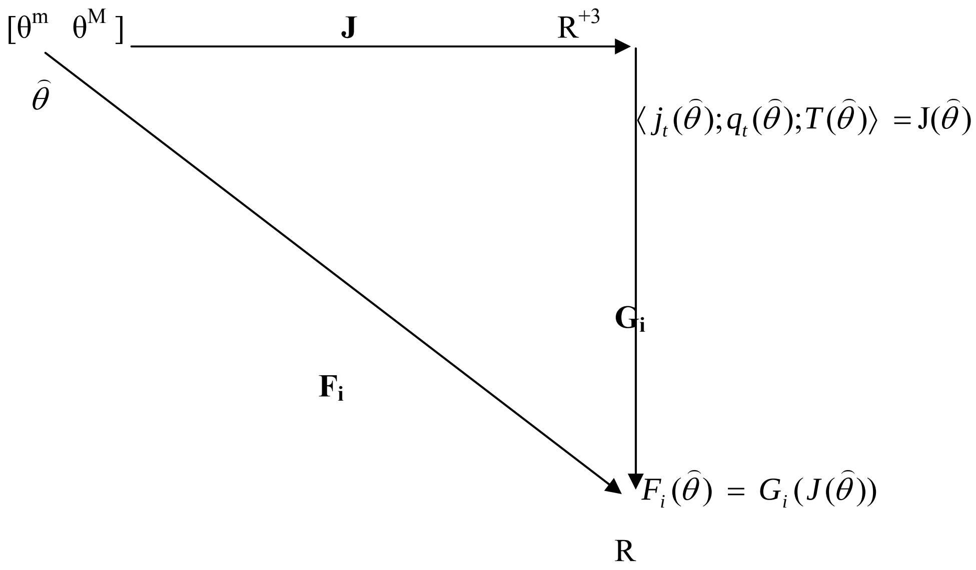
Let J be a function defined from R^{+} into R^{+3} such as:

$$J : [\theta^{m}, \theta^{M}] \longrightarrow R^{+3}$$

Such that for each report $\hat{\theta}$ in $[\theta^{m}, \theta^{M}]$ revealed by the *Maj'uul* on the cost efficiency is associated a triplet $\langle j_t(\hat{\theta}); q_t(\hat{\theta}); T(\hat{\theta})\rangle$ of positive real numbers in R^{+3}

representing the mechanism offered by the *Ja'eel* where $j_t(\hat{\theta})$; $q_t(\hat{\theta})$, and $T(\hat{\theta})$ represent respectively the monetary transfer by the *Ja'eel* to the *Maj'uul* at the end of each period t , the quantity extracted in period t and the time at which the contract ends. The *Ja'eel* is now supposed to determine the three elements of the triplet that maximize his objective function.

The objective function of the *Ja'eel* and *Maj'uul* may be characterized by the following diagram.



The function F_i is defined as the composition of the functions G_i and J for i = 1,2 . (The *Maj'uul* and the *Ja'eel*.).

In the discrete case the objective functions of the *Maj'uul* and the *Ja'eel* take the following forms :

$$F_1(\hat{\theta}) = (G_1 \circ J)(\hat{\theta}) = G_1(J(\hat{\theta})) = -K_0 + \sum_{t=1}^{T(\hat{\theta})} \frac{1}{(1+\beta_1)^t} [\, j_t(\hat{\theta}) - C(\theta, q_t, S_t)] \qquad (2)$$

$$F_2(\hat{\theta}) = (G_2 \circ J)(\hat{\theta}) = G_2(J(\hat{\theta})) = \sum_{t=1}^{T(\hat{\theta})} \frac{1}{(1+\beta_2)^t} [\, p q_t(\hat{\theta}) - j(\hat{\theta})] \qquad (3)$$

The expression between brackets in equation (2) represents the difference between the *Ju'ul* (remuneration) that the *Maj'uul* gets from the *Ja'eel* and his total cost. His total cost is equal to the cost of extraction plus the cost of depletion of the mine per period .In other words, the expression between brackets represents the net gain of the *Maj'uul* per period of time. β_1 and K_0 represent respectively the

Maj'uul's time preference and the amount of capital needed by the *Maj'uul* to finance the extraction operation during the first period. The expression between brackets in equation (3) denotes the net benefits per period accruing to the *Ja'eel*. It is equal to the difference between the revenues accruing from selling the output (q) at the market price (p) and the *Ju'ul* (remuneration as a fraction of the output given to the *Maj'uul*). β_2 represents the time preference of the *Ja'eel*.

In the continuous case Equation (2) and Equation (3) become :

$$F_1(\theta) = - K_0 + \int_{t_0}^{T(\hat{\theta})} e^{-\beta_1 t} [\, j(\hat{\theta}) - C(\theta, q_t, S_t)] \, dt \qquad (4)$$

$$F_2(\hat{\theta}) = \int_0^{T(\hat{\theta})} e^{-\beta_2 t} [\, pq_t - j(\hat{\theta})] \, dt \qquad (5)$$

The Jae' el will then maximize Equation (5) subject to the following constraint s :

$$\dot{S}_t = - q_t \qquad \text{for } t = 0 \ , \ S_t = S_0 \qquad (6)$$

$$\Pi_{Mj} = \int_0^{T} e^{-\beta_1 t} [j(\hat{\theta}) - C(\theta, q_t, S_t)] dt - K_0 \geq 0 \qquad (7)$$

$$\int_0^{T} q_t(\hat{\theta}) \, dt \leq S_0 \qquad (8)$$

Equations (6), (7) and (8) represent the constraints that the *Ja'eel* will be facing when maximizing his objective function. Equation (6) says that the rate of depletion of the mine-stock is equal in absolute value to the rate of extraction. In other words, the level of the stock of the mine decreases at a rate equal to the extraction rate.

K_0 represents the amount of capital needed by the *Maj'uul* at the beginning of period one to cover the cost of the extraction during that period. The *Ja'eel* is not supposed to make any advance payment. The function C(θ, q_t, S_t) denotes the total cost of the *Maj'uul*. The general form of the cost function includes the cost of extraction, the cost of depletion and the cost of exploration. However, in our model we will consider only the two first costs, i.e., the cost of extraction and depletion since the exploration phase is assumed to be terminated. $J_t(\theta)$ represents the *Ju'ul* determined by the *Ja'eel* on the basis of the report θ given by the *Maj'uul* which represents the efficiency cost parameter, fully known only to the Maj'ul in case of asymmetric information. However, in case of symmetric information, both parties know the true value of θ and hence θ will be equal to $\hat{\theta}$.

A financier like an Islamic bank, may join the *Maj'uul* in financing the cost of period one by signing a new contract between the *Maj'uul* and the financier .The model may then be extended to include three parties, the *Ja'eel*, the *Maj'uul* and the financier. The contract between the *Maj'uul* and the Islamic bank (the financier) may take the form of *Musharaka*[12], *Mudharaba*[13], *Murabaha*[14], or *Istisna'*[15]. In the case of *Murabaha*, the *Maj'uul* will approach the financier to finance the purchase of equipment needed for the extraction operation on a *Murabaha* basis. In the case of *Istisna'*, the financier may advance money needed by the *Maj'uul* to finance his operation on the basis of an *Istisna'* contract where the financier plays the role of *Mustasna'*[16] (the person who would be requisitioning the specific product to be manufactured) and the *Maj'uul* the role of *Sana'* (manufacturer). To avoid to be stuck with the merchandise purchased through thee first *Istisna'* contract and to stick to its main role as financial intermediary the bank contracts a parallel *Istisna'* with clients in need of the product

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12 *Musharaka* is a relationship established by the parties through a mutual contract.

13 *Mudharaba* is a special kind of partnership where one partner gives money to another for investing it in a commercial enterprise. The investment comes from the first partner who is called "***Rab-ul-Mal***", while the management and work is an exclusive responsibility of the other ,who is called **"*Mudharib*"** .

14 *Murabaha* is in fact, a term of Islamic Fiqh and it refers to a particular kind of sale .If a seller agrees with his purchaser to provide him a specific commodity on a certain profit added to his cost ,it is called "*murabaha"* transaction. The basic ingredient of "*murabaha*" is that the seller discloses the actual cost he has incurred in acquiring the commodity, and then adds some profit.

15 *Istisna'* is a sale where a commodity is transacted before it comes into existence. This means to order a manufacturer to manufacture a specific commodity for the purchaser. If the manufacturer undertakes the fabrication of the goods for him with material from the manufacturer, the transaction of *Istisna'* comes into existence. It is necessary for the validity of *Istisna'* that the price be fixed with the consent of the parties and that necessary specifications of the commodity intended to be manufactured is fully settled between them. The contract creates therefore a moral obligation on the manufacturer to manufacture the goods, but before he starts the work, anyone of the two parties may cancel the contract after giving a notice to the other. However, after the manufacturer has started the work, the contract cannot be cancelled.

16 Mustasna' is the person who would be requisitioning the specific product to be manufactured.

acquired in the first *Istisna‘* contract. In this latter case, the financier will play the role of *Sana‘* (manufacturer)and the clients the role of *Mustasna‘*(the person who would be requisitioning the specific product to be manufactured). In this framework, the financier is playing the dual role of a *Sana’* and *Mustasna‘*.

3.3 The *Maj’uul*

Let the *Maj’uul*'s costs at period to be given by some function $C(q_t,\theta,S_t)$ where q_t represents the extraction rate of the mineral at time t, S_t the stock of the mine at time t and θ the firm's efficiency cost parameter.

For the sake of simplicity and clarity we assume that the *Maj’uul*'s cost function $C(q_t, \theta,S_t)$ at period t takes the following form[17]:

$$C(q_t, \theta,S_t) = \theta q_t^2 + S_0 - \gamma S_t \quad \text{if } q_t \text{ is greater zero} \qquad (9)$$
$$= 0 \quad \text{if } q_t = 0$$

The total cost is equal to the sum of the cost of extraction ($\theta q^2{}_t$) and the cost of depletion ($S_0 - \gamma S_t$). The parameter γ is of geological nature and is supposed to be known to everyone. The parameter θ is considered private information and is known only to the *Maj’uul* in case of asymmetric information; however, in this paper it is a known information to all.

Let $F(\theta)$ denotes the cumulative distribution of θ, defined on the closed interval $[\theta^m,\theta^M]$. To this distribution is associated a probability density function $f(\theta)$ assumed to be differentiable on the open interval $]\theta^m, \theta^M[$. The cumulative distribution function is a common knowledge to both the *Ja’eel* and *Maj’uul*.

[17] Several other forms of cost functions have been used .Gaudet ,Lasserre (1995) and Long used a quadratic cost function in q_t ; Poudou and Thomas (2000) used a cost function that has two parts. The first part of the cost function represents the extraction cost while the second part represents the cost for depletion . The first part of the cost function is a quadratic form in q_t .The second part is linear with respect to the stock of the mine at time t.

Applying feature 5 of Section 2.3, the *Ju'ul* can be written as a fraction of the extracted output q_t. Therefore, we define the *Ju'ul* schedule J_t for all t to be equal to:

$$J_t = j_t p q_t \qquad \text{for} \quad t \in [\,0,T\,] \tag{10}$$

Where J_t and j_t represent respectively the monetary transfer from the *Ja'eel* (principal) to the *Maj'uul* (agent) and the *Maj'uul*'s agreed share as a fraction of the output extracted in period t. The coefficient j_t takes values between 0 and 1.

Finally, the *Maj'uul* is subject to the following dynamic constraint that governs the depletion of the stock of the mine over time.

$$\dot{S}_t(\hat{\theta}) = -q_t(\hat{\theta}) \tag{11}$$

3.4 The *Ja'eel*

Based on assumption A1, the *Ja'eel* maximizes the expected social welfare which consists of the present value of profits from the extraction operation over the whole period [0,T][18]. That is

$$\text{Max } W = \int_0^{T(\hat{\theta})} e^{-\beta_2 t}\,[\,pq_t(\hat{\theta}) - j(\hat{\theta})]\,dt \tag{12}$$

Subject to the following constraints :

$$\dot{S}(\hat{\theta}) = -q_t(\hat{\theta}) \tag{13}$$

$$\Pi_{Mj} = \int_0^{T} e^{-\beta_1 t}\,[j(\hat{\theta}) - C(\theta, q_t, S_t)\,]dt - K_0 \geq 0 \tag{14}$$

$$\int_0^{T} q_t(\hat{\theta})\,dt \leq S_0 \tag{15}$$

[18] In this model, the *Ja'eel* plays the role of the state. Therefore, he will be maximizing the social welfare function which consists of the present value of profits from the extraction operations over the period. However, the *Ja'eel* may play the role of the principal in an agency set-up.

Substituting Equations (9) and (10) into the system of Equations (12)-(15) the above maximization problem becomes

$$\text{Max } W = \int_0^{T(\hat{\theta})} (1 - j)\, e^{-\beta_2 t}\, p\, q_t(\hat{\theta})\, dt$$

Subject to the following constraints (16)

$$\dot{S}_t(\hat{\theta}) = -q_t(\hat{\theta})$$

$$\Pi_{Mj} = \int_0^{T} e^{-\beta_1 t} [jp\, q_t(\hat{\theta}) - \theta\, q^2{}_t - S_0 + \gamma S_t] dt - K_0 \geq 0$$

$$\int_0^{T} q_t(\hat{\theta})\, dt \leq S_0$$

Where j_t is considered as constant . The above maximization problem is equivalent to the following optimization problem (see appendix):

$$\max W = \int_0^{T} e^{-\beta_2 t} (pq - \theta q^2 - S_0 + \gamma S)\, dt$$

subject to (17)

$$\dot{S}_t = -q$$

$$S_0(\theta) = S_0$$

The maximization of the objective function of the *Ja'eel* as expressed by equation (17) will determine the trajectory of the extraction rate over the agreed period [0,T] based on the furnished report by the *Maj'uul* .Two situations are distinguished, the case of symmetric information and the case of asymmetry of information. We shall only treat the first case in details. We shall give some indications in the concluding part of the study how assumptions A5 and A7 may be relaxed. These two assumptions relate respectively to the exploration-extraction case and the possibility of relaxing the assumption of the exogenous property of the price of the mineral .

4
DERIVATION OF THE OPTIMALITY CONDITIONS AND ECONOMIC INTERPRETATION OF THE MODEL UNDER SYMMETRIC INFORMATION

In the symmetric information case both the *Ja'eel* and the *Maj'uul* observe the parameter θ which becomes now common knowledge. The problem of the *Ja'eel* becomes to maximize the first Equation in the system (16) subject to the constraints associated with. The third constraint in (16) is equivalent to the boundary condition S(0) = S_0. Moreover, the current value Hamiltonian, H, corresponding to Equation (17), is written as:

$$H(\theta) = e^{-\beta_2 t}\left(pq_t - \theta q_t^2 - S_0 + \gamma S\right) - \lambda_t(\theta) q_t(\theta) \qquad (18)$$

Where the multiplier λ expresses the marginal contribution of the stock of the mine prevailing at time t to the social welfare when discounted back to the initial period. Among the necessary conditions for maximizing society's welfare are the "optimality conditions":

$$-\gamma e^{-\beta_2 t} = \dot{\lambda} \qquad (19)$$

$$\dot{S} = -q \qquad (20)$$

$$e^{-\beta_2 t}(p - 2\theta q) - \lambda \le 0 \qquad (21)$$

$$q[e^{-\beta_2 t}(p - 2\theta q) - \lambda] = 0 \qquad (22)$$

with the transversality condition:

$$\lambda_T(\theta) S_T(\theta) = 0 \qquad (23)$$

The solution of the above system of equations gives the path of the extraction rate of the mineral that maximizes the social welfare .This trajectory of the extraction rate of the mineral is given by the following equation. See Equation (18) Appendix I for derivation.

$$q_t(\theta) = \frac{1}{2\theta}\{\ p - e^{\beta_2 t}\lambda_T + \frac{\gamma}{\beta_2}[e^{-\beta_2(T-t)} - 1]\ \} \qquad (24)$$

Equation (24) shows that the extraction $q_t(\theta)$ depends on the cost efficiency parameter of the *Maj'uul* θ, the social planner's time preference rate[19] β_2, the market price of the mineral, the parameter γ which is of geological nature and on λ(θ). It is clear from the same equation that the extraction rate is positively related to the price of the mineral, and inversely related to the *Ja'eel*'s time preference rate β_2 assuming that the stock of the mineral is not depleted at the end of the extraction period . These results are in conformity with what we expected because as the price of mineral increases the *Ja'eel* supplies more mineral expecting therefore the extraction rate to increase to meet the growing demand .On the other hand, as the time preference rate of the *Ja'eel* increases the accruing expected discounted benefits decrease leading to an adverse effect on the extraction rate .

From the transversality condition given by Equation (23) we distinguish two cases, either $S_T(\theta) = 0$ or $S_T(\theta) > 0$.

Case No.1 $S_T(\theta) > 0$

The first case implies that some minerals are left over after the agreed period [0,T]. From the transversality condition, $S_T(\theta) > 0$ implies that λ_T (θ) must equal zero. Equation (24) becomes:

[19] β_2 is taken to be the social planner's time preference whenever the *Ja'eel* is considered to be the state which will try to maximize the welfare function. However, if the *Ja'eel* plays the role of the entrepreneur ,he will maximize his expected utility function.

$$q_t(\theta) = \frac{1}{2\theta}\{p - \frac{\gamma}{\beta_2} + \frac{\gamma}{\beta_2} e^{\beta_2(t-T)}\} \tag{25}$$

let A denotes the expression between brackets. Totally differentiating Equation (25) we get $\mathrm{dq} = -\frac{\mathrm{A}}{2\theta^2} d\theta + \frac{\mathrm{dA}}{2\theta}$

where dA is equal to

$$\mathrm{dA} = \mathrm{dp} + \frac{e^{-\beta_2(T-t)}}{\beta_2}\mathrm{d}\gamma + \frac{\gamma\left\{1 - e^{-\beta_2(T-t)}[1+\beta_2(T-t)]\right\}}{\beta_2^2}\mathrm{d}\beta_2 \tag{26}$$

Substituting equation (26) into the previous equation we get the total differential of equation (25).

$$\mathrm{dq} = -\frac{\mathrm{A}}{2\theta^2} d\theta + \frac{1}{2\theta}\{\mathrm{dp} + \frac{e^{-\beta_2(T-t)}}{\beta_2}\mathrm{d}\gamma + \frac{\gamma\left\{1 - e^{-\beta_2(T-t)}[1+\beta_2(T-t)]\right\}}{\beta_2^2}\mathrm{d}\beta_2\ \} \tag{27}$$

From Equation (26) it is clear that the expression denoted by A is positively related to the price and to the parameter γ which is of geological nature. On the hand simple calculation shows that the expression between brackets in the numerator of the *Ja'eel*'s time preference rate β_2 is negative since $e^{-\beta_2(T-t)}$ is greater than unity making its inverse less than unity whereas the expression $[1 + \beta_2(T-t)]$ is greater than unity.

Equation (27) shows that as the cost efficiency parameter teta increases the extraction rate q set up by the *Ja'eel* decreases holding the other terms constant. Moreover, Equation (27) shows that the extraction rate q is positively related to both the exogenous price p of the mineral and the parameter γ which is of geological nature. However, as the *Ja'eel*'s time preference rate β_2 increases the extraction rate q decreases since the expression between brackets of the factor d β_2 in the numerator is negative. Having determined the path of the extraction rate of the *Ja'eel* in terms of the time preference of the *Maj'uul* θ we would like to know the value of this latter. The value of θ is given by equation (28) below as derived in Appendix I and shown by Equation (19).

$$\theta_c \quad = \quad \frac{\text{A(T)}}{2\text{S}_0} \qquad (28)$$

where

$$\text{A(T)} \;=\; p(T) - \frac{\gamma}{\beta_2}T + \frac{\gamma}{\beta_2{}^2}(1 - e^{-\beta_2 T})$$

Equation (28) shows that the value of the cost efficiency parameter θ of the *Maj'uul* is positively related to the price of the mineral p, and inversely related to the initial stock as well as to the *Ja'eel* 's time preference rate β_2 . Therefore for each θ will correspond an extraction rate q.

Case No2. $S_T(\theta) = 0$

S_T equals to zero implies that the leftover is no more economically profitable. Hence, from the transversality condition , $\lambda_T(\theta)$ must be positive. This latter is given by Equation (22) in Appendix I. Equation (22) is reproduced below for convenience .

$$\lambda_T(\theta) \quad = \quad \frac{2\text{S}_0 \beta_2}{\text{e}^{\beta_2 T} - 1} \; \{ \theta_\text{c} \; - \; \theta \} \qquad (29)$$

Equation (29) shows the existence of a critical value for the efficiency cost parameter of the *Maj'uul* θ which is given by Equation (30) below and derived in Appendix I and reproduced here for convenience .

$$\theta_c \quad = \quad \frac{A(T)}{2S_0} \qquad (30)$$

where A(T) has been defined before.

It is clear from Equation (30) and from the definition of A(T) that the critical value depends on the geological nature γ, the time period T, the initial stock S_0, the time preference rate of the *Ja'eel* β_2, and the market price of the mineral p. Hence, any change in these parameters affects negatively or positively the critical value of the efficiency cost parameter of the *Maj'uul* θ_c which in its turn affects the optimal extraction rate q(θ). Simple calculations as shown in Appendix II show that the critical value θ_c is negatively related to the initial stock S_0 and positively related to the market price p, but has no definite sign as the time period T, the geological nature parameter γ and the time preference rate of the *Ja'eel* β_2 change. However, regarding the change in the critical value of the efficiency cost parameter of the *Maj'uul* θ_c with respect to the time preference rate of the *Ja'eel* β_2 we have some partial results which say that the critical value of the efficiency cost parameter increases as the time preference rate of the *Ja'eel* increases as long as (T β_2 -2) is non negative[20] .

Now substituting the value of $\lambda_T(\theta)$ as given by Equation (29) in Equation (24) above we get Equation (31) below for the trajectory of the extraction q_t .

$$q_t(\theta) = \frac{1}{2\theta}\{p_t - \frac{\gamma}{\beta_2} + \frac{\beta_2 e^{\beta_2 t}}{(e^{\beta_2 T} - 1)}[\, 2\theta S_0 - \mathrm{p(T)} + \frac{\gamma}{\beta_2}\mathrm{T}]\ \} \tag{31}$$

We observe that as θ moves from its minimum value θ_m to its maximum value θ_M, θ passes through a critical value θ_c .The optimal extraction rate q will then take two forms depending whether the value of the efficiency parameter θ is inferior or superior to the critical value θ_c . The two forms of the optimal extraction rate are given below

- If $\theta_m < \theta < \theta_c$

The optimal extraction rate takes the form

$$q_t(\theta) = \frac{1}{2\theta}\{p - \frac{\gamma}{\beta_2} + \frac{\gamma}{\beta_2} e^{\beta_2 (t-T)}\} \tag{32}$$

- If $\theta_c < \theta < \theta_M$

[20] See Appendix 2 for more details.

The optimal extraction rate is equal to

$$q_t(\theta) = \frac{1}{2\theta}\{p_t - \frac{\gamma}{\beta_2} + \frac{\beta_2 e^{\beta_2 t}}{(e^{\beta_2 T} - 1)}[\,2\theta S_0 - p(T) + \frac{\gamma}{\beta_2} T]\ \} \qquad (33)$$

In case of symmetric information, the optimal contract leads to the existence of a critical value of the efficiency cost parameter of the *Maj'uul* which as long as the resource is depleted, its stock in situ diminishes and the extraction cost rises owing to the fact that it becomes more and more difficult to dig. This stock effect introduces the possibility for the *Maj'uul* to learn the level of in situ reserves and may keep it as a private information which raises the problem of asymmetric information.

We conclude that as the parameters change, the critical value of the cost efficiency parameter of the *Maj'uul* θ_c also varies.

5
CONCLUDING REMARKS

Mining sector is one of the most important sectors of the economy as it serves as an engine for the economic growth of most Muslim economies. Its exploration and exploitation need large investments and advanced technology that most of the Muslim countries lack. The call for foreign companies to invest in this sector is not without a cost. Concession contracts have always been in favor of the foreign firms that try to rip the maximum wealth of these countries. In this paper we studied the *Ja'ala* contract as an alternative for the existing concession contracts.

This paper shows how the *Ja'ala* contract may be used to exploit the mining sector. First, the study provides an overview of the *Shari'ah* principles that govern the *Ja'ala* contract. It lays down the main features of the *Ja'ala* contract and discusses its potentiality of how it could be used as a base for financial intermediation with some comparative analysis with other Islamic modes of financing. Second, The study gives a full characterization of the *Ja'ala* contract for the exploitation model in both cases - in discrete time as well as in continuous time - that can be easily extended to the exploration and exploitation case. The study gives a rigorous treatment on how the bilateral relationship between the *Ja'eel* and *Maj'uul* is developed. It shows the interrelationship between the cost efficiency parameter of the *Maj'uul* and the extraction rate of the mineral imposed by the *Ja'eel*. It derives the path of the extraction rate which is found to depend on several variables, namely, the efficiency cost parameter of the *Maj'uul*, the social planner's time preference rate, the market price of the mineral, and the marginal contribution of the stock of the mine prevailing at the time to the social welfare when discounted back to the initial period. Finally, the study derives a critical value for the efficiency cost parameter of the *Maj'uul* corresponding to the case where the leftover of the mineral after the agreed period is no more profitable.

The study may be extended in at least two ways. One possible extension is to extend the model to include an exploration phase. One major incentive for adding the exploration phase into the model is the downward impact on extraction costs. This can be done as follows: in the cost function of the *Maj'uul* a cost of an exploratory effort may be added. Having introduced that, the change in the stock of the mine will then depend on two factors: the extraction rate and the discovery rate. In addition, the discovery rate will also be depending on exploratory effort as well. Another possible extension is to relax the assumption of the price of the mineral which is assumed to be exogenous. This can be explained by the existence of the negative influence of the exploration on market price of the mineral. Hence, the equilibrium price path will depend on whether the price of the mineral is greater or less than the marginal cost of discoveries. Taking this into the model, the price of the mineral can no more be taken as an exogenous variable.

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APPENDIX I

Let us consider the following maximization problem

$$W = \int_0^T e^{-\beta_2 t}(1 - j)\, pq\, dt \qquad (1)$$

subject to

$$\pi_{MJ}(\theta) = \int_0^T e^{-\beta_1 t}(\, jpq - \theta q^2 - S_0 + \gamma S)\, dt \ - K_0 \ \geq 0 \qquad (2)$$

$$\dot{S} = - q_t \qquad (3)$$

$$S_t = S_0 - \int_0^t q_z(\theta)\, dz \qquad \text{and} \qquad S_0(\theta) = S_0 \qquad (4)$$

Let $\pi_t = jpq - \theta q^2 - S_0 + \gamma S \geq 0$ denotes the profit of the Maj'uul at instant t with the assumption of perfect competition. Simple calculation shows that

$$(1 - j)pq = pq - \theta q^2 - S_0 + \gamma S - \pi_t \qquad (5)$$

Substituting Equation (5) into Equation (1) and using the definition of the profit Π(θ) Equation (1) can be written as:

$$W = \int_0^T e^{-\beta_2 t}(\, pq - \theta q^2 - S_0 + \gamma S)\, dt \ - \left\{ \int_0^T e^{-\beta_2 t}\, \pi_t dt - K_0 \right\} - \ K_0 \qquad (6)$$

Hence

$$W = \int_0^T e^{-\beta_2 t}(\, pq - \theta q^2 - S_0 + \gamma S)\, dt \ - \ \pi(\theta) \ - \ K_0 \qquad (7)$$

We assume the profit Π (θ) of the *Maj'uul* to be equal to zero. The maximization problem given by Equation (1) becomes then:

$$W = \int_0^T e^{-\beta_2 t} \left(pq - \theta q^2 - S_0 + \gamma S \right) dt \quad - \quad K_0 \tag{8}$$

Maximizing W is equivalent to maximizing $W + K_0$. The initial maximization problem becomes therefore,

$$\text{Max } W = \int_0^T e^{-\beta_2 t} \left(pq - \theta q^2 - S_0 + \gamma S \right) dt$$

Subject to (9)

$$\dot{S} = -q$$

$$S_0(\theta) = S_0$$

$$q_t \geq 0 \text{ for all } t \quad \text{and } p_t \geq 0 \text{ for all } t$$

The current-value Hamiltonian, H, corresponding to Equation (9), is written as

$$H(\theta) = e^{-\beta_2 t} \left(pq - \theta q^2 - S_0 + \gamma S \right) - \lambda_t(\theta) q_t(\theta) \tag{10}$$

Where the multiplier $\lambda_t(\theta)$ expresses the marginal contribution of the stock of the mine at time t to domestic welfare when discounted back to the initial period.

The necessary conditions for maximizing society's welfare using straight forward control theory are as follows. The variables S and q denote respectively the state variable and the control variables.

$$-\frac{\partial H}{\partial S} = \dot{\lambda} \qquad \Leftrightarrow \qquad -\gamma e^{-\beta_2 t} = \dot{\lambda} \qquad (11)$$

$$\frac{\partial \mathrm{H}}{\partial \lambda} = \dot{\mathrm{S}} \qquad \Leftrightarrow \qquad \dot{S} = -q \qquad (12)$$

$$\frac{\partial H}{\partial q} \leq 0 \qquad \Leftrightarrow \qquad e^{-\beta_2 t}(p - 2\theta q) - \lambda \leq 0 \qquad (13)$$

$$q\frac{\partial H}{\partial q} = 0 \qquad \Leftrightarrow \qquad q[e^{-\beta_2 t}(p - 2\theta q) - \lambda] = 0 \qquad (14)$$

with the transversality condition

$$\lambda_T(\theta) S_T(\theta) = 0 \qquad (15)$$

Let us now integrate Equation (1)

$\int_t^T d\lambda = -\gamma \int_t^T e^{-\beta_2 S} dS$ the integration implies $\lambda_T - \lambda_t = \frac{\gamma}{\beta_2} e^{-\beta_2 S} \Big|_t^T$

$$\lambda_T - \lambda_t = \frac{\gamma}{\beta_2}[e^{-\beta_2 T} - e^{-\beta_2 t}]$$

Which gives the following relationship between λ_t *and* λ_T

$$\lambda_t = \lambda_T - \frac{\gamma}{\beta_2}[e^{-\beta_2 T} - e^{-\beta_2 t}] \qquad (16)$$

Substituting Equation (16) into Equation (14) we obtain the trajectory .

$$e^{-\beta_2 t}(p - 2\theta q) - \lambda_t = 0$$
$$p - 2\theta\, q = \lambda e^{\beta_2 t}$$
$$2\theta\, q = p - \lambda e^{\beta_2 t}$$
$$q \;=\; \frac{1}{2\theta}[p - \lambda e^{\beta_2 t}] \qquad (17)$$

By substituting λ_t in Equation (17) we obtain the traject or y $q_t(\theta)$ (an interior solution) which governs the extraction of the mine over time.

The extraction rate of the mine is set up by the Ja'eel to maximize the welfare.

The Maj'uul will implement the rate of extraction given to him by the Ja'eel.

$$q_t(\theta) = \frac{1}{2\theta}[p - \{\lambda_T - \frac{\gamma}{\beta_2}(e^{-\beta_2 T} - e^{-\beta_2 t})\}e^{\beta_2 t}]$$
$$q_t(\theta) = \frac{1}{2\theta}\{p - \lambda_T e^{\beta_2 t} + \frac{\gamma}{\beta_2}e^{\beta_2(t-T)} - \frac{\gamma}{\beta_2}\}$$

Rearranging the terms we get

$$q_t(\theta) = \frac{1}{2\theta}\{p - e^{\beta_2 t}\lambda_T + \frac{\gamma}{\beta_2}(e^{-\beta_2(T-t)} - 1)\} \qquad (18)$$

Equation (18) represents the path of the extraction rate over time. It tells that at every moment the *Ja'eel* is able to determine the rate of extraction which will be given to the *Maj'uul* to implement based on the report furnished by the *Maj'uul* .

From the transversality conditions given above in Equation (15) we have two cases. The first case represents the situation when some minerals are left over after the agreed period of extraction, whereas the second case deals with the situation of no mineral leftover.

$Case$ No 1 : $S_T(\theta) > 0$

$Since$ $S_T(\theta)$ is greater th an zero then $\lambda_T(\theta)$ must equal zero .

In this case Equation (18) becomes

$$q_t(\theta) = \frac{1}{2\theta}\{p - \frac{\gamma}{\beta_2} + \frac{\gamma}{\beta_2} e^{\beta_2(t-T)}\}$$

Now having determined the path of the extraction rate in terms of θ, we use the initial conditions to find the value of θ .

$$S_T(\theta) = S_0 - \int_0^T q_t(\theta)dt$$

$$S_T(\theta) = S_0 - \frac{1}{2\theta}\int_0^T (p - \frac{\gamma}{\beta_2} + \frac{\gamma}{\beta_2} e^{\beta_2(t-T)})dt$$

$$S_T(\theta) = S_0 - \frac{1}{2\theta}[p(T) - \frac{\gamma}{\beta_2}T + \frac{\gamma}{\beta^2{}_2} e^{-\beta_2 T}\int_0^T e^{\beta_2 t}dt$$

$$where \ \ \mathrm{p(T)} = \int_0^T \mathrm{p}_t dt$$

$$S_T(\theta) = S_0 - \frac{1}{2\theta}[p(T) - \frac{\gamma}{\beta_2}T + \frac{\gamma}{\beta_2{}^2}(1 - e^{-\beta_2 T})]$$

$$let \ \ \mathrm{A(T)} = p(T) - \frac{\gamma}{\beta_2}T + \frac{\gamma}{\beta_2{}^2}(1 - e^{-\beta_2 T})$$

the above equation becomes

$$\mathrm{S_T} = \mathrm{S_0} - \frac{1}{\theta}A(T)$$

Solving the above equation with respect to θ we have:

$$\theta_c \quad = \quad \frac{\text{A(T)}}{2\text{S}_0} \qquad (19)$$

Equation (19) shows that θ is inversely related to the initial stock and positively related to the price of the mineral in particular.

Case No 2 : $S_T(\theta) = 0$

$$S_T(\theta) = 0 \text{ implies that } \quad \lambda_T(\theta) \; \geq \; 0$$

$$S_T(\theta) = 0 \Leftrightarrow S_0 - \int_0^T q_t(\theta)dt = 0$$

$$S_0 = \int_0^T q_t(\theta)dt = \frac{1}{2\theta}\int_0^T (p - \frac{\gamma}{\beta_2} - e^{\beta_2 t}\lambda_T + \frac{\gamma}{\beta_2}e^{-\beta_2 T}e^{\beta_2 t})_{dt}$$

$$S_0 = \frac{1}{2\theta}\int_0^T [p + \; + \frac{\gamma}{\beta_2}(e^{-\beta_2(T-t)} - 1) \;]dt \; - \frac{\lambda_T}{2\theta}\int_0^T e^{\beta_2 t}dt$$

Hence

$$S_0 = \frac{1}{2\theta}\left\{ A(T) - \frac{1}{2\theta\beta_2}\lambda_T (e^{\beta_2 T} - 1)\right\}$$

where $A(T)$ is defined as before to be equal to

$$A(T) = \int_0^T [\, p + \; + \frac{\gamma}{\beta_2}(e^{-\beta_2(T-t)} - 1) \;]dt$$

with simple manipulati on we get the solution for λ_T

$$\lambda_T = \frac{2S_0\beta_2}{(e^{\beta_2 T} - 1)}[\frac{A(T)}{2S_0} - \theta\,] \qquad (20)$$

Let θ_c represents the first term within brackets of Equation(20) then the above equation may be put in the following form

$$\lambda_T(\theta) \; = \; \frac{2\text{S}_0\beta_2}{\text{e}^{\beta_2 T} - 1} \; \{\theta_c \; - \; \theta\} \qquad (21)$$

From Equation (21) we may conclude that there exist a critical value for θ equal to θ_c for which λ_T (θ) is greater than zero. This critical value has been defined to be equal to the first term of the expression between brackets in Equation (20).

Now substituting the value of $\lambda_T(\theta)$ into Equation (18) and after some simple manipulations we get the value for the trajectory of the rate of extraction of the mineral $q_t(\theta)$.

$$q_t(\theta) = \frac{1}{2\theta}\{p_t - \frac{\gamma}{\beta_2} + \frac{\beta_2 e^{\beta_2 t}}{(e^{\beta_2 T} - 1)}[\,2\theta S_0 - \mathrm{p(T)} + \frac{\gamma}{\beta_2}\mathrm{T}]\,\} \qquad (22)$$

Equation (22) shows that the extraction rate is inversely related to the cost efficiency parameter of the *Maj'uul* in this second case.

APPENDIX 2

We shall study the changes of the critical value of the efficiency cost parameter θ_c with respect to the changes in the time preference rate of the *Ja'eel* β_2, the parameter γ of geological nature, the price of the mineral p, the time period T and the initial stock S_o .

For this let us reproduce Equation(30) below.

$$\theta_c = \frac{\gamma(1-e^{-\beta_2 T})}{2\beta_2^2 S_0} + \frac{(p-\frac{\gamma}{\beta_2})T}{2S_0} \quad (1)$$

Simple manipulations of Equation(1) give the following equation

$$\theta_c = \frac{\gamma}{2\beta_2^2 S_0} + \frac{(p-\frac{\gamma}{\beta_2})T}{2S_0} - \frac{\gamma}{2\beta_2^2 S_0 e^{\beta_2 T}} \quad (2)$$

1. Change in critical value of the efficiency cost parameter θ_c with respect to the geological nature parameter γ : $\frac{d\theta_c}{d\gamma}$

From Equation(2) we have

$$\frac{d\theta_c}{d\gamma} = \frac{T}{2\beta_2 S_o} - \frac{1}{2\beta^2{}_2 S_o} - \frac{1}{2\beta^2{}_2 S_o e^{\beta_2 T}} \quad (3)$$

which can be written as

$$\frac{d\theta_c}{d\gamma} = \frac{1}{2\beta_2 S_o}[\,T - \frac{1}{\beta_2}(1 + e^{-\beta_2 T})\,] \quad (4)$$

It is clear from the expression on the right-hand side of Equation (4) may be positive or negative depending on whether the value $\beta_2 T$ is greater or less than 1+ $e^{-\beta_2 T}$. Therefore, as the parameter of geological nature increases the critical value of the cost efficiency parameter of the *Maj'uul* is ambiguous.

2. $\frac{d\theta_c}{dS_o}$: Change in critical value of the efficiency cost parameter θ_c with respect to the Change in the initial Stock S_o

It is clear from Equation(1) that the cost efficiency parameter of the *Maj'uul* is negatively related to the initial stock of the mineral S_o since this latter is contained in the denominator of the terms of the right-hand side of Equation (1).

3. $\frac{d\theta_c}{dp}$ Change in critical value of the efficiency cost parameter θ_c with respect to the change in the price of the mineral p.

From Equation (1) above simple calculations show that

$$\frac{d\theta_c}{dp} = \frac{T}{2S_o} \geq 0$$

Hence as the price of the mineral increases the critical value of the cost efficiency parameter of the *Maj'uul* increases.

4. $\frac{d\theta_c}{dT}$: Change in critical value of the efficiency cost parameter θ_c with respect to the change in the time period T

From Equation (2) above we have

$$\frac{d\theta_c}{dT} = \frac{p - \frac{\gamma}{\beta_2}}{2S_o} + \frac{\gamma\beta_2}{2S_o\beta_2^{\,2}} e^{-\beta_2 T}$$

After some manipulations this latter equation can be written as follows:

$$\frac{d\theta_c}{dT} = \frac{1}{2S_o}[\,p + \frac{\gamma}{\beta_2}(e^{-\beta_2 T} - 1)\,] \qquad (5)$$

It is clear from Equation (5) above that the expression between parentheses is negative. Hence the expression between brackets may then take positive or negative values depending on whether the price of the mineral p is greater or inferior to the second term between brackets.

5. $\frac{d\theta_c}{d\beta_2}$ Change in critical value of the efficiency cost parameter θ_c with respect to the change in the time preference rate of the *Ja'eel* β_2.

From Equation (2) above we have

$$\frac{d\theta_c}{d\beta_2} = \frac{T\gamma}{2S_o\beta_2^2} - \frac{\gamma}{S_o\beta_2^3} + \frac{\gamma}{2S_o}\frac{(\beta_2^2 T\, e^{-\beta_2 T} + 2\beta_2 e^{-\beta_2 T})}{\beta_2^4}$$

Simple manipulation leads

$$\frac{d\theta_c}{d\beta_2} = \frac{\gamma}{2S_o\beta_2^2}[\,T - \frac{1}{\beta_2} + \frac{1}{\beta_2}(\,T\beta_2 + 2)e^{-\beta_2 T}\,] \qquad (6)$$

Equation (6) may also be written as

$$\frac{d\theta_c}{d\beta_2} = \frac{\gamma}{2S_o\beta_2^3}[\,(\,T\beta_2 + 2)e^{-\beta_2 T} - 2 + T\beta_2\,]$$

which also can be written in the following form

$$\frac{d\theta_c}{d\beta_2} = \frac{\gamma}{2S_o\beta_2^3 e^{\beta_2 T}}[(T\beta_2 - 2)e^{\beta_2 T} + (T\beta_2 + 2)] \quad (7)$$

Equation shows that as the time preference rate of the *Ja'eel* β_2 increases the critical value of the cost efficiency parameter θ_c may increase or decrease depending on whether the value of the first term between brackets $(T\beta_2 - 2)e^{\beta_2 T}$ is positive or negative, i.e., whether the expression $T\beta_2 - 2$ is greater or less than zero. If it is greater than zero this implies that both terms within brackets are positive and therefore as the time preference rate of the *Ja'eel* increases, the cost efficiency parameter increases too. On the other hand, if the expression $T\beta_2 - 2$ is negative this will make the first term within brackets negative making the sign of the whole expression on the right-hand side of Equation (7) above, ambiguous.