M. Fahim Khan

Macro-Consumption Function in an Islamic Framework
JRIE Vol.1, No.2, Winter 1404/1984

Comments: Seif E.I. Tagel-Din

Department of Economics
University of Khartoum Sudan

The paper by Dr. M. Fahim Khan has truly been a genuine attempt to combine mathematical formulation and computer simulation of a purely behavioral model. Nevertheless, the basic mathematical axioms of the model, which the author has derived on purely intuitive grounds, do not seem consistent with the object of the analysis. My discussion of these problems is stated below.

The Paradox of "Taqwa" and Spending In the way of Allah: The author has based his analysis on the utility function:

$$U = U(E_1, E_2) \tag{1}$$

where E_1 = Consumer's own worldly spending
and E_2 = That part of spending which he makes in the way of Allah
E_1 and E_2 should be treated as real commodities rather than money values, so that we should write the income-constraint as

$$Y = P_1 E_1 + P_2 E_2 \tag{2}$$

instead of $Y = E_1 + E_2$ as the author has done. The coefficients P_1 and P_2 are the corresponding prices per unit of E_1 and E_2 respectively. The usual Diminishing Marginal Utility assumptions are being expressed for E_1 as

$$U_{E1} > 0 \qquad \text{and} \quad U_{E1\,E1} < 0 \tag{3}$$

Yet, the component E_2 which is spent in the way of Allah required a different set of assumptions in the author's view:

$$U_{E2} = a > 0 => U_{E2\,E2} = 0 \tag{4}$$

where "a" is a given constant. Now, if we combine (3) and (4) we should be able to write:

$$U(E_1, E_2) = U(E_1) + a E_2 \tag{5}$$

Thus a separability condition is imposed on the utility function due to the fact that an independent *cardinal* utility model has been derived for E_2.

Now, if we fix $U = U_0$ at a given curve in two-dimensional utility space we should get:
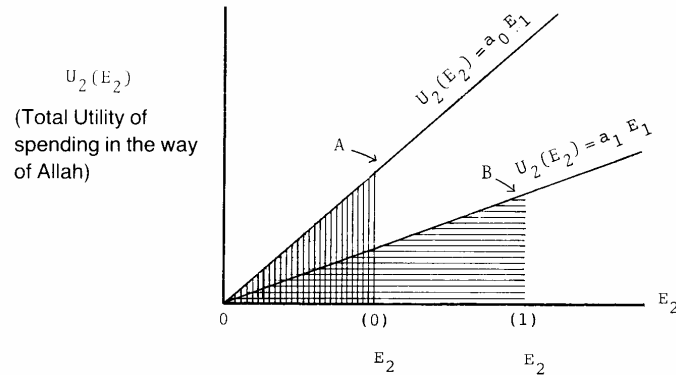
$$E_2 = \frac{1}{a}\left(U_0 - U_1(E)\right) \qquad (6)$$

leading to:

$$\frac{dE}{da}2 = \frac{-1}{a^2}\left(U_0 - U_1(E_1)\right) \qquad (7)$$

and since $a > 0$, $U_0 - U_1(E_1) > 0$, it follows that $\frac{dE}{da}2 < 0$ contrary to $U1(E1)$

what the author has derived!

A more intuitive illustration of this point can be made with reference to the geometrical diagram below:

The two total utility curves $a_0 E_1$ and $a_1 E_1$ are drawn such that $a_0 > a_1$. The consumer with larger "a"($= a_0$) spends $E_2^{(o)}$ units to enjoy a total utility represented by the triangle $(o, E_2^{(o)} A)$ But, in order for the second consumer with the smaller "a" ($=a$) to enjoy the same level, he must spend additional units of E_2 equal to $E_2^{(1)} - E_2^{(o)} > 0$ so that the second triangle $(0, E_2^{(1)}, B)$ is equal to the first Hence, it is obvious that:

$$a_0 > a_1 ==> E_2^{(o)} < E_2^{(1)} \qquad (8)$$

just in line with (7) above.

In words: this result implies that a higher level of *taqwa*, measured by the parameter "a" in the author's view, is compatible with a smaller rather than a bigger amount of spending in the way of Allah. This paradox is brought more clearly in focus if we note that:

Lim { T (a) } = 0

a   (9)

where T (a) is the inverse of the author's relation= F (T) assumed single-valued; i.e. T(a)= level of *taqwa* with degree "a". This limiting statement implies that extreme positive levels of *taqwa* are associated with "spending almost nothing" in the way of Allah! And as "spending nothing" is compatible with non-believers, then the highest level of *taqwa* are very close to their secular opposite!!

The convexity Paradox: The indifference curve approach implies that there is a set of various combinations $((E_1, E_2))$ which yield the consumer the same level of total utility, $U = U_0$ The total derivative of the indifference curve at this fixed level is:

$$d\ U_0 = U_{E1}\ U_{E1}\ dE_1 + a\ dE_2 = 0 \tag{10}$$

leading to:

$$\frac{dE_2}{dE_1} = \frac{-U_{E1}}{a} < 0 \tag{11}$$

since $a > 0$ and $U_{E1} > 0$.

This first condition implies that the indifference curve is downward sloping as usual. Then to examine the convexity property of the curve we look at the second condition of the maximum-assuming as usual that the consumer is a utility maximizer.

The income-constraint leads to:

$$E_2 = \frac{Y - P_1 E_1}{P_2} = b - CE_1 > 0$$

where $b = \dfrac{Y}{b}$ and $C = \dfrac{P_1}{P_2}$. Then, we are effectively maximizing

$$U_0 = U_1\ (E_1) + a\ (b - C\ E_1) \tag{12}$$

The convexity property, which is the second condition of the maximum, is satisfied if

$$\frac{d^2 U_0}{dE^2} < 0 \tag{13}$$

and is fulfilled since $U_{E1}\ E_1 < 0$. Thus, our indifference curve is down-ward sloping and convex to the origin.

However, this is another paradox which implies that a Muslim consumer must reduce his spending in the way of Allah, E_2, by systematically equal amounts while increasing his own worldly consumption, E_1, by systematically bigger and bigger amounts, and this process keep him at the same level of satisfaction! This is only compatible with those who are trading off the after-life rewards for worldly pleasures!! Of course this is contrary to what the author desires to establish. Clearly the constant marginal utility of E_2 does not help. Moreover, there is some problem with the prices P_1 and P_2 which apparently reflect two different real commodities, not of the same quality! For, if the Muslim consumer loves for his poor brother what he loves for himself the two commodities should be identical and the indifference curve approach fails.

Some Other Points: Apart from the above-mentioned technical problems, the relevance of the basic individualistic indifference curve approach to Islamic consumers' behavior is questionable. It is possible in principle to make use of the new extensions in utility theory which work with general utility models involving interdependent co-operative groups (e.g. the family); see Becker, Lan caster, Muth, or Samuelson. Such an approach should be more relevant for capturing the collectivist nature of Muslim communities, if properly adapted.

Finally, I may question the statement made by the author that *zakah* receivers would dynamically be promoted to *zakah* givers. I believe that this property to a great extent depends on the relative magnitude of the "Substitution effect" in the underlying labor supply curve as the "income effect" in this case is equal to zero, if not negative. In the latter case the income-effect may reinforce the substitution of more leisure for work as more work may imply less income from *zakah*. Hence, this point needs to be treated with special care in future research.

Conclusion

The basic utility model taken as a point of departure in the "Macro-consumption Function..." does not really represent the behavior of a genuine Muslim consumer. First it implies that higher levels of *taqwa* are inconsistent with spending in the way of Allah. Second, it describes an unaltruistic person who is ready to trade-off after-life reward for worldly pleasures. Finally, it does not allow for the collectivist nature of Muslim communities.

References

Becker, G.S., "A Theory of the Allocation of Time", *The Economic Journal*, Vol. **75**, p. 493. 1965.

Khan M. Fahim, "Macro-Consumption Function in an Islamic Framework", *Journal of Research in Islamic Economics*, Vol. **1**, No. **2**, winter, 1984.

Lancaster, K., "The Theory of Household Behavior", *Annual of Economic and Social Measurement*, Vol. **4**, No.**1** & **5**, 1975.

Muth, R.F., "Household Production and Consumer Demand Function", *Econometrica*, Vol. **34**, No. **3**, p.699, 1966.

Samuelson P., "Social Indifference Curves", *Quarterly Journal of Economics*, Vol. **70**, No.**1**, 1956.